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PHILOSOPHICAL TRANSACTIONS.

XV.—*Researches in Physical Astronomy.* By J. W. LUBBOCK, Esq. V.P. and
Treas. R.S.

Read May 19, 1831.

On the Theory of the Moon.

THE method pursued by CLAIRAUT in the solution of this important problem of Physical Astronomy, consists in the integration of the differential equations furnished by the principles of dynamics, upon the hypothesis that in the gravitation of the celestial bodies the force varies inversely as the square of the distance, and in which the true longitude of the moon is the independent variable; the time is thus obtained in terms of the true longitude, and by the reversion of series the longitude is afterwards obtained in terms of the time, which is necessary for the purpose of forming astronomical tables. But while on the one hand this method possesses the advantage, that the disturbing function can be developed with somewhat greater facility in terms of the true longitude of the moon than in terms of the mean longitude, yet on the other hand, the differential equations in which the true longitude is the independent variable are far more complicated than those in which the time is the independent variable. The latter equations are used in the planetary theory; so that the method of CLAIRAUT has the additional inconvenience, that while the lunar theory is a particular case of the problem of the three bodies, one system of equations is used in this case, and another in the case of the planets.

The method of CLAIRAUT has been adopted, however, by MAYER, by LAPLACE, and by M. DAMOISEAU. The last-mentioned author has arranged his results with remarkable clearness, so that any part of his processes may be easily verified by any one who does not shrink from this gigantic undertaking; and the immense labour which this method requires, when all sensible quantities

are retained, may be seen in his invaluable memoir. Mr. BRICE BRONWIN has recently communicated to the Society a lunar theory, in which the same method is adopted.

Having reflected much upon the difficulties of this problem, I am led to believe that the integration of the differential equations in which the time is the independent variable, is at least as easy as the method hitherto, I think, solely employed, and I now have the honour to submit to the Society a lunar theory founded upon this integration, which is in fact merely an extension of the equations given in my *Researches in Physical Astronomy*, already printed, by embracing those terms which, in consequence of the magnitude of the eccentricity of the moon's orbit, are sensible; and the suppression of those, on the other hand, which are insensible on account of the great distance of the sun, the disturbing body. By means of the Table which I have given (Table II.), the developments may all be effected at once with the greatest facility.

The first column contains the indices, which I have employed to distinguish the inequalities. The numbers in the second column are the indices affixed by M. DAMOISEAU, in the *Mém. sur la Théor. de la Lune*, p. 547. to the inequalities of longitude.

$$t^* = nt - n_1 t, \quad x = cnt - \omega, \quad z = n_1 t - \omega, \quad y = gnt - \nu.$$

0	..	0	21	45	$2t - 3x$	42	73	$2t - 3x - z$
1	30	$2t \dagger$	22	46	$2t + 3x$	43	..	$2t + 3x + z$
2	1	x	23	21	$2x + z$	44	26	$3x - z$
3	31	$2t - x \ddagger$	24	53	$2t - 2x - z$	45	..	$2t - 3x + z$
4	32	$2t + x$	25	54	$2t + 2x + z$	46	..	$2t + 3x - z$
5	16	$z \S$	26	20	$2x - z$	47	..	$2x + 2z$
6	33	$2t - z$	27	51	$2t - 2x + z$	48	75	$2t - 2x - 2z$
7	34	$2t + z$	28	52	$2t + 2x - z$	49	..	$2t + 2x + 2z$
8	2	$2x$	29	23	$x + 2z$	50	..	$2x - 2z$
9	35	$2t - 2x$	30	59	$2t - x - 2z$	51	..	$2t - 2x + 2z$
10	36	$2t + 2x$	31	..	$2t + x + 2z$	52	..	$2t + 2x - 2z$
11	19	$x + z$	32	22	$x - 2z$	53	..	$x + 3z$
12	41	$2t - x - z$	33	61	$2t - x + 2z$	54	..	$2t - x - 3z$
13	42	$2t + x + z$	34	60	$2t + x - 2z$	55	..	$2t + x + 3z$
14	18	$x - z$	35	..	$3z$	56	..	$x - 3z$
15	39	$2t - x + z$	36	..	$2t - 3z$	57	..	$2t - x + 3z$
16	40	$2t + x - z$	37	..	$2t + 3z$	58	..	$2t + x - 3z$
17	17	$2z$	38	9	$4x$	59	..	$4z$
18	43	$2t - 2z$	39	67	$2t - 4x$	60	..	$2t - 4z$
19	44	$2t + 2z$	40	..	$2t + 4x$	61	..	$2t + 4z$
20	4	$3x$	41	27	$3x + z$	62	3	$2y$

* Inconvenience arises from using the letter t in this acceptance. I have done so in order to conform to the notation of M. DAMOISEAU. $\dagger$ Variation. $\ddagger$ Evection. $\S$ Annual Equation.

63	37	$2t - 2y$	105	84	$t + z$	146		y
64	38	$2t + 2y$	106	85	$t - 2x$	147		$2t - y$
65	5	$x - 2y$	107	86	$t + 2x$	148		$2t + y$
66	6	$x + 2y$	108	91	$t - x - z$	149		$x - y$
67	49	$2t - x - 2y$	109	92	$t + x + z$	150		$x + y$
68	47	$2t - x + 2y$	110	89	$t - x + z$	151		$2t - x - y$
69	48	$2t + x - 2y$	111		$t + x - z$	152		$2t - x + y$
70	50	$2t + x + 2y$	112		$t - 2z$	153		$2t + x - y$
71	24	$z - 2y$	113		$t + 2z$	154		$2t + x + y$
72	25	$z + 2y$	114		$t - 2y$	155		$z - y$
73	57	$2t - z - 2y$	115		$t + 2y$	156		$z + y$
74	56	$2t - z + 2y$	116	100	$3t$	157		$2t - z - y$
75	55	$2t + z - 2y$	117	101	$3t - x$	158		$2t - z + y$
76	58	$2t + z + 2y$	118	102	$3t + x$	159		$2t + z - y$
77	7	$2x - 2y$	119	103	$3t - z$	160		$2t + z + y$
78	8	$2x + 2y$	120	104	$3t + z$	161		$2x - y$
79	65	$2t - 2x - 2y$	121		$3t - 2x$	162		$2x + y$
80	63	$2t - 2x + 2y$	122		$3t + 2x$	163		$2t - 2x - y$
81	64	$2t + 2x - 2y$	123		$3t - x - z$	164		$2t - 2x + y$
82	..	$2t + 2x + 2y$	124		$3t + x + z$	165		$2t + 2x - y$
83	..	$x + z - 2y$	125		$3t - x + z$	166		$2t + 2x + y$
84	..	$x + z + 2y$	126		$3t + x - z$	167		$x + z - y$
85	..	$2t - x - z - 2y$	127		$3t - 2z$	168		$x + z + y$
86	..	$2t - x - z + 2y$	128		$3t + 2z$	169		$2t - x - z - y$
87	..	$2t + x + z - 2y$	129		$3t - 2y$	170		$2t - x - z + y$
88	..	$2t + x + z + 2y$	130		$3t + 2y$	171		$2t + x + z - y$
89	..	$x - z - 2y$	131	120	$4t$	172		$2t + x + z + y$
90	..	$x - z + 2y$	132	121	$4t - x$	173		$x - z - y$
91	..	$2t - x + z - 2y$	133	122	$4t + x$	174		$x - z + y$
92	..	$2t - x + z + 2y$	134	123	$4t - z$	175		$2t - x + z - y$
93	..	$2t + x - z - 2y$	135	124	$4t + z$	176		$2t - x + z + y$
94	..	$2t + x - z + 2y$	136	125	$4t - 2x$	177		$2t + x - z - y$
95	..	$2z - 2y$	137	126	$4t + 2x$	178		$2t + x - z + y$
96	..	$2z + 2y$	138	131	$4t - x - z$	179		$2z - y$
97	..	$2t - 2z - 2y$	139		$4t + x + z$	180		$2z + y$
98	..	$2t - 2z + 2y$	140	129	$4t - x + z$	181		$2t - 2z - y$
99	..	$2t + 2z - 2y$	141		$4t + x - z$	182		$2t - 2z + y$
100	..	$2t + 2z + 2y$	142		$4t - 2z$	183		$2t + 2z - y$
101	80	t^*	143		$4t + 2z$	184		$2t + 2z + y$
102	81	$t - x$	144	127	$4t - 2y$	185		$t - y$
103	82	$t + x$	145		$4t + 2y$	186		$t + y$
104	83	$t - z$						

$$\cos 2t \cos 2t = \frac{1}{2} \cos 4t + \frac{1}{2} \quad [131] \quad [0]$$

$$\cos 2t \cos x = \frac{1}{2} \cos (2t + x) + \frac{1}{2} \cos (-2t + x) \quad [4] \quad [-3]$$

Hence the multiplication of $\cos 2t$ by $\cos 2t$ produces the arguments 131 and 0, similarly the multiplication of $\cos x$ by $\cos 2t$ produces the arguments 4 and -3 ; proceeding in this way the following Table was formed, by writing down the indices instead of the arguments themselves.

* Parallaxic inequality.

TABLE I.

Showing the arguments which result from the combination of the arguments 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 17, 20, 35, 62, 101, 146 and 147, with the arguments 1, 2, 3, &c. by addition and subtraction.

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
1 {	131 0	4 3	132 2	133 -2	7 6	134 5	135 -5	10 9	136 8	137 -8	13 12	138 11	16 15	19 18	22 21	37 36	64 63	116 101	148 147	 146	1
2 {	4 -3	8 0	1 -9	10 -1	11 14	16 -12	13 -15	20 -2	3 -21	22 -4	23 -5	6 -24	26 5	29 32	38 -8	53 56	66 65	103 -102	150 149	153 -151	2
3 {	132 -2	1 9	136 0	131 -8	15 12	138 -14	140 -11	4 21	 2	133 -20	7 24	 5	6 27	33 30	10 39	57 54	68 67	117 102	152 151	 -149	3
4 {	133 2	10 1	131 8	137 0	13 16	141 11	139 14	22 3	132 20	 -2	25 6	134 23	28 7	31 34	40 9	55 58	70 69	118 103	154 153	 150	4
5 {	7 -6	11 -14	15 -12	13 -16	17 0	1 -18	19 -1	23 -26	27 -24	25 -28	29 -2	3 -30	2 -32	35 -5	41 -44	59 -17	72 71	105 -104	156 155	159 -157	5
6 {	134 -5	16 12	138 14	141 -11	1 18	142 0	131 -17	28 24	 26	 -23	4 30	 2	34 3	7 36	46 42	19 60	74 73	119 104	158 157	 -155	6
7 {	135 5	13 15	140 11	139 -14	19 1	131 17	143 0	25 27	 23	 -26	31 3	132 33	4 33	37 6	43 45	61 18	76 75	120 105	160 159	 156	7
8 {	10 -9	20 2	2 -21	22 -3	23 26	28 -24	25 27	38 0	1 -39	 -1	41 14	16 -42	44 11	47 50			78 77	107 -106	162 161	165 -163	8
9 {	136 -8	3 21	 -2	132 -20	27 24	 -26	 -23	1 39	 0	131 -38	15 42	 -14	12 45	51 48			80 79	121 106	164 163	 -161	9
10 {	137 8	22 4	133 20	 2	25 28	 23	 26	40 1	131 38	 0	43 16	141 41	46 13	49 52			82 81	122 107	166 165	 162	10
11 {	13 -12	23 5	7 -24	25 -6	29 2	4 -30	31 -3	41 -14	15 -42	43 -16	47 0	1 -48	8 17	53 14			84 83	109 -108	168 167	171 -169	11
12 {	138 -11	6 24	 -5	134 -23	3 30	 -2	132 -29	16 42	 14	141 -41	1 48	 0	18 9	15 54			86 85	123 108	170 169	 -167	12
13 {	139 11	25 7	135 23	 5	31 4	133 29	 2	43 15	140 41	 -14	49 1	131 47	10 19	55 16			88 87	124 109	172 171	 168	13
14 {	16 -15	26 -5	6 -27	28 -7	2 32	34 -3	4 -33	44 -11	12 -45	46 -13	8 -17	18 -9	50 0	11 56			90 89	111 -110	174 173	177 -175	14
15 {	140 -14	7 27	 5	125 -26	33 3	132 -32	 -2	13 45	 11	139 -44	19 9	136 17	1 51	57 12			92 91	125 110	176 175	 -173	15
16 {	141 14	28 6	134 26	 -5	4 34	 2	133 32	46 12	138 44	 -11	10 18	142 8	52 1	13 58			94 93	126 111	178 177	 174	16
17 {	19 -18	29 -32	33 -30	31 34	35 5	7 -36	37 -6	47 -50	51 -48	49 -52	53 -14	15 -54	11 -56	59 0			96 95	113 -112	180 179	183 -181	17
18 {	142 -17	34 30	 32	 -29	6 36	 -5	134 -35	52 48	 50	 -47	16 54	 14	58 12	1 60			92 97	127 112	182 181	 -179	18
19 {	143 17	31 33	 29	 -32	37 7	135 35	 5	49 51	 47	 -50	55 15	140 53	13 57	61 1			100 99	128 113	184 183	 180	19
20 {	22 -21	38 8	10 -39	40 -9	41 44																20
21 {	 -20	9 39	 -8	136 -38	45 42																21

TABLE I. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147		
70 {	 66	82 64	145 78	 62	88 94																70	
71 {	75 - 74	83 90	91 86	87 94	95 62																71	
72 {	76 - 73	84 89	92 85	88 93	96 62																72	
73 {	 - 72	93 85	 89	 84	63 97																73	
74 {	 - 71	94 86	 90	 83	64 98																74	
75 {	 71	87 91	 83	 90	99 63																75	
76 {	 72	88 92	 84	 89	100 64																76	
101 {	116 -101	103 102	117 -102	118 -103	105 104	119 -104	120 -105	107 106	121 -106	122 -107	109 108	123 -108	111 110	113 112			115 114	1	186 0	185	 -185	101
102 {	117 -103	101 106	121 -101	116 -107	110 108																102	
103 {	118 -102	107 101	116 -106	122 -101	109 111																103	
104 {	119 -105	111 108	123 -110	126 -109	101 112																104	
105 {	120 -104	109 110	125 -108	124 -111	113 101																105	
116 {	 101	118 117	 103	 102	120 119	 105	 104	122 121	 107	 106	124 123	 109	126 125	128 127			130 129	131 1		 186	116	
117 {	 102	116 121	 101	 106	125 123																117	
118 {	 103	122 116	 107	 101	124 126																118	
119 {	 104	126 123	 111	 108	116 127																119	
120 {	 105	124 125	 109	 110	128 116																120	
131 {	 1	133 132	 4	 3	135 134	 7	 6	137 136	 10	 9	139 138	 13	141 140	143 142			145 144	 116		 148	131	
132 {	 3	131 136	 1	 9	140 138																132	
133 {	 4	137 131	 10	 1	139 141																133	
134 {	 6	141 138	 16	 12	131 142																134	
135 {	 7	139 140	 13	 15	143 131																135	
146 {	148 -147	150 -149	152 -151	154 -153	156 -155	158 -157	160 -159	162 -161	164 163	166 -165	168 -167	170 -169	174 -173	180 -179	...	...		186 -185	62 0	 - 63	146	
147 {	 -146	153 151	 149	 -150	159 157	 155	 -156	164 163	 161	 -162	171 169	 167	177 175	183 181	...	...	148	 185	1 63	144 0	147	

TABLE I. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
148 {	 146	154 152	 150	 -149	160 158	 156	 -155	166 164	 162	 -161	172 170	 168	 178 176	184 182	...	...	 147	 186	64 1	131 62	} 148
149 {	153 -152	161 -146	147 -164	165 -148	167 173																} 149
150 {	154 -151	162 146	148 -163	166 -147	168 174																} 150
151 {	 -150	147 163	 -146	 -162	175 169																} 151
152 {	 -149	148 164	 146	 -161	176 170																} 152
153 {	 149	165 147	 161	 -146	171 177																} 153
154 {	 150	166 148	 162	 146	172 178																} 154
155 {	159 -158	167 -174	175 -170	 -178	179 -146																} 155
156 {	160 -157	168 -173	176 -169	172 -177	180 146																} 156
157 {	 -156	177 169	 173	 -168	147 181																} 157
158 {	 -155	178 170	 174	 -167	148 182																} 158
159 {	 155	171 175	 167	 -174	183 147																} 159
160 {	 156	172 176	 168	 -173	184 148																} 160

	38	59	
1	{ 40 39	{ 61 60	1

TABLE II.

Showing the arguments which, by their combination with the arguments 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 17, 20, 35, 62, 101, 146, 147, produce the arguments, 12, 3, &c. in the left hand column. This Table is formed from the preceding, by making the numbers in the left hand column in that Table change places with the rest. A full stop is placed after the figure where it does not occupy the same *cell* as in the preceding Table.

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147		
1 {	0 131	3 4	2 132	133. - 2	6 7	5	 - 5	9 10	8 ...	 - 8	12 13	11	15 16	18 19	...	...	63 64	101 116	147 148	146 ...	}	1
2 {	4. - 3	0 8	- 9. 1	10. - 1	14 11	16. - 12	- 15. 13	 - 2	 3	 - 4	 - 5	 6	5 ...		...	...	...	...	...	...	}	2
3 {	132. - 2	9 1	0	131. - 8	12 15	 - 14	 - 11	 4	2 ...		 7	 5	 6		...	...	...	...	...	...	}	3
4 {	2 133	1 10	8 131	0	16 13	11	14	3	 - 2		6	 7			...	...	...	...	...	...	}	4
5 {	7. - 6	- 14. 11	15. - 12	- 16. 13	0 17	- 18. 1	19. - 1				 - 2	 3	 2	 - 5	...	...	...	...	...	...	}	5
6 {	134. - 5	12 16	14	 - 11	18 1	 - 17	131.				 4	 2	3	 7	...	...	...	...	...	...	}	6
7 {	5 135	15 13	11	 - 14	1 19	17 131					3	 4	 6		...	...	...	...	...	...	}	7
8 {	10. - 9	2 20	- 21. 4	22. - 3	26 23				 1	 - 1	 14	 16	11 ...	 - 2	...	...	...	...	...	...	}	8
9 {	 - 8	21 3	 - 2	132. - 20	24 27			 1	 131	 15	 - 14	12	 4		...	...	...	...	...	...	}	9
10 {	8	4 22	20 133	2	28 25			1	 131	 16	 13	 3			...	...	...	...	...	...	}	10
11 {	13. - 12	5 23	- 24. 7	25. - 6	2 29	 4	 - 3	 - 14	 15	 - 16		 1	 17	14 8		...	...	...	...	...	}	11
12 {	 - 11	24 6	 - 5	134. - 23	30 3	 - 2		 16	 14		 1	 9	 18	 15		...	...	...	...	...	}	12
13 {	11	7 25	23 135	5	4 31		2	15	 - 14		1 131	 19	 10	 16		...	...	...	...	...	}	13
14 {	16. - 15	26. - 5	- 27. 6	28. - 7	32 2	 - 3	 4	 - 11	 12	 - 13	- 17. 8	18. - 9	 11		...	...	...	...	...	...	}	14
15 {	 - 14	27 7	5	135. - 26	3 33	 - 2	 13	 11		 9	 17	 12	 1		...	...	...	...	...	...	}	15
16 {	14	6 28	26 134	 - 5	34 4	2		12	 - 11	 18	8 10	1	 13		...	...	...	...	...	...	}	16
17 {	19. - 18	- 32. 29	33 - 30	- 34. 31	5 35	 7	 - 6			 - 14	 15	 11	 0		...	 - 5					}	17
18 {	 - 17	30 34	32	 - 29	36 6	 - 5				 16	 14	12	 1		 7						}	18
19 {	17	33 31	29	 - 32	7 37		5			 15	 13	 1		 6							}	19
20 {	22. - 21	8	 10	 - 9				2	 4	 - 3					 0						}	20

TABLE II. (Continued.)

1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
21 {	 - 20	 9	 - 8				 3	 - 2						 1						
22 {	 20	 10		 8			 4		 2					 1						
23 {	 - 25. - 24	 11	 13	 - 12	 8	 10	 - 9	 5	 7	 - 6	 2	 4								
24 {	 - 23	 12	 - 11		 9	 - 8		 6	 - 5		 3	 - 2								
25 {	 23	 13		 11	 10		 8	 7		 5	 4									
26 {	 - 28. - 27	 14	 16	 - 15	 8	 - 9	 10	 - 5	 6	 - 7		 2								
27 {	 - 26	 15	 - 14		 9		 8	 7	 5			 3								
28 {	 26	 16		 14	 10	 8		 6		 - 5		 4								
29 {	 - 31. - 30	 17	 19	 - 18	 11	 13	 - 12				 5	 7		 2						
30 {	 - 29	 18	 - 17		 12	 - 11					 6	 - 5		 3						
31 {	 29	 19		 17	 13		 11				 7			 4						
32 {	 - 34. - 33	 - 17	 18	 - 19	 14	 - 15	 16					 - 5	 2							
33 {	 - 32	 19		 17	 15		 14					 7	 3							
34 {	 32	 18		 17	 16	 14						 6	 4							
35 {	 - 37. - 36			 17	 19	 - 18							 5							
36 {	 - 35			 18	 - 17								 6		 1					
37 {	 35			 19		 17							 7		 1					
38 {		 20	 22	 - 21			 8	 10	 - 9					 2						
39 {		 21	 - 20				 9	 - 8						 3						
40 {		 22		 20			 10							 4						
41 {		 23	 25	 - 24	 20		 11	 13	 - 12	 8	 10			 5						
42 {		 24	 - 23		 21		 12	 - 11		 9	 - 8			 6						
43 {		 25		 23	 22		 13		 11	 10				 7						
44 {		 26	 28	 - 27	 20		 14	 16	 - 15			 8		 - 5						

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
45 {					21	...	...				...			9	...	7	...	...	...	...	45
		27	- 26			...	...	15	- 14		...				...						
46 {		28		26		...	...	16		14	...		10	...	6	...	...	...	...	...	46
					22	...	...				...				...						
47 {		29		23	...	...	...	17			11			8	...	...	...	...	...	...	47
			31	- 30		...	...		19	- 18	...	13			...						
48 {		30	- 29		24	...	...	18	- 17		12	- 11		9	...	...	...	...	...	...	48
						...	...				...				...						
49 {		31		29	25	...	...	19		17	13			10	...	...	...	...	...	...	49
						...	...				...				...						
50 {		32			26	...	...		18	- 19	...		14	...	8	...	...	...	...	...	50
			34	- 33		...	...	- 17			...				...						
51 {		33	- 32		27	...	...		17		...			9	...	...	...	...	...	...	51
						...	...	19			...		15	...	...	...	...	...	...	...	
52 {		34		32		...	...	18			...		16	...	10	...	...	...	...	...	52
					28	...	...			- 17	...				...						
53 {		35		29	...	...	...				17			11	...	2	...	...	...	...	53
			37	- 36		...	...				...	19			...						
54 {		36	- 35		30	...	...				18	- 17		12	...	3	...	...	...	...	54
						...	...				...				...						
55 {		37		31	...	...	...				19			13	...	4	...	...	...	...	55
						...	...				...				...						
56 {			36	- 37	32	...	...				...			14	...	2	...	...	...	...	56
		- 35				...	...				...		- 17								
57 {			35		33	...	...				...			15	...	3	...	...	...	...	57
		37				...	...				...		19	...	...	...	...	...	...	...	
58 {		36		35		...	...				...		18	...	...	...	...	...	...	...	58
				- 35	34	...	...				...			16	...	4	...	...	...	...	
59 {					35	...	...				...			17	...	5	...	...	...	...	59
						...	...				...				...						
60 {					36	...	...				...			18	...	6	...	...	...	...	60
						...	...				...				...						
61 {					37	...	...				...			19	...	7	...	...	...	...	61
						...	...				...				...						
62 {	- 64.	66.	68.	70.	72.	...	...				...			...	...	...	...	146	...	...	62
	- 63	- 65	- 67	- 69	- 71	...	...				...				...			...	148	...	
63 {		67	65		73	...	...				...			...	...	...	1	...	147	- 146	63
	- 62	69		- 66	75	...	...				...				...			...			
64 {	62	68	66		74	...	...				...			...	...	...	1	148	...	...	64
		70		- 65	76	...	...				...				...			...			
65 {	- 69.					...	...				...			...	...	...	2	...	...	...	65
	- 68	- 62	63	- 64		...	...				...				...			...			
66 {	- 70.	62				...	...				...			...	...	...	2	...	...	...	66
	- 67		64	- 63		...	...				...				...			...			
67 {	- 76					...	...				...			...	...	...	3	...	...	...	67
	- 66	63	- 62			...	...				...				...			...			
68 {			62			...	...				...			...	...	...	3	...	...	...	68
	- 65	64				...	...				...				...			...			

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
69 {	65	63	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	69
	...	...	...	62	...	...	...	...	...	...	...	...	...	...	...	...	4	...	...	...	
70 {	66	64	...	62	...	...	...	...	...	...	...	...	...	...	...	...	4	...	...	...	70
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
71 {	75.	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	71
	- 74	...	...	...	- 62	63	- 64	...	...	...	...	...	...	...	...	...	5	...	...	...	
72 {	76.	...	...	...	62	...	...	...	...	...	...	...	...	...	...	...	5	...	...	...	72
	- 73	...	...	...	...	64	- 63	...	...	...	...	...	...	...	...	...	...	...	...	...	
73 {	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	73
	- 72	...	...	...	63	- 62	...	...	...	...	...	...	...	...	...	...	6	...	...	...	
74 {	...	...	...	...	...	62	...	...	...	...	...	...	...	...	...	...	...	...	...	...	74
	- 71	...	...	...	64	...	...	...	...	...	...	...	...	...	...	...	6	...	...	...	
75 {	71	...	...	...	63	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	75
	...	...	...	...	...	...	- 62	...	...	...	...	...	...	...	...	...	7	...	...	...	
76 {	72	...	...	...	64	...	62	...	...	...	...	...	...	...	...	...	...	...	...	...	76
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	7	...	...	...	
77 {	...	65	69.	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	77
	...	65	- 68	...	...	...	...	- 62	63	- 64	...	...	...	...	...	...	8	...	...	...	
78 {	...	66	...	...	...	...	...	62	...	...	...	...	...	...	...	...	8	...	...	...	78
	...	70	- 67	...	...	...	...	64	- 63	...	...	...	...	...	...	...	...	...	...	...	
79 {	...	67	- 66	...	...	...	...	63	- 62	...	...	...	...	...	...	...	9	...	...	...	79
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
80 {	...	68	...	...	...	...	...	64	62	...	...	...	...	...	...	...	9	...	...	...	80
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
81 {	...	69	...	65	...	...	...	63	...	- 62	...	...	...	...	...	...	10	...	...	...	81
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
82 {	...	70	...	66	...	...	...	64	62	...	...	...	...	...	...	...	10	...	...	...	82
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
83 {	...	71	...	...	65	...	...	...	...	- 62	63	...	...	...	...	...	11	...	...	...	83
	...	75	- 74	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
84 {	...	72	...	...	66	...	...	...	...	62	...	...	...	...	...	...	11	...	...	...	84
	...	76	- 73	...	...	...	...	...	...	...	64	...	...	...	...	...	...	...	...	...	
85 {	...	...	...	...	67	...	...	...	...	...	63	- 62	...	...	...	...	12	...	...	...	85
	...	73	- 72	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
86 {	...	...	...	...	68	...	...	...	...	...	64	...	...	...	...	...	12	...	...	...	86
	...	74	- 71	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
87 {	...	75	...	71	69	...	...	...	...	...	63	...	...	...	...	...	13	...	...	...	87
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
88 {	...	76	...	72	70	...	...	...	...	...	64	...	...	...	...	...	13	...	...	...	88
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
89 {	...	...	...	...	65	...	...	...	...	...	...	...	- 62	...	...	...	14	...	...	...	89
	...	- 72	73	- 76	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
90 {	...	...	...	...	66	...	...	...	...	...	...	...	62	...	...	...	14	...	...	...	90
	...	- 71	74	- 75	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
91 {	...	...	71	...	67	...	...	...	...	...	...	...	...	...	...	...	15	...	...	...	91
	...	75	...	...	...	...	...	...	...	...	...	...	63	...	...	...	...	...	...	...	
92 {	...	...	...	...	68	...	...	...	...	...	...	...	...	...	...	...	15	...	...	...	92
	...	76	...	...	...	...	...	...	...	...	...	...	64	...	...	...	...	...	...	...	

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
93 {		73			69			...			...		63		...	...	16		...	...	93
94 {		74			70			...			...		64		...	...	16		...	...	94
95 {		...			71			...			...		...	62	...	...	17		...	...	95
96 {		...			72			...			...		...	62	...	...	17		...	...	96
97 {		...			73			...			...		...	63	...	...	18		...	...	97
98 {		...			74			...			...		...	64	...	...	18		...	...	98
99 {		...			75			...			...		...	63	...	...	19		...	...	99
100 {		...			76			...			...		...	64	...	...	19		...	...	100
101 {	116. -101	102 103	117. -102	118. -103	104 105			...			...		...		...	...	...	1		...	101
102 {	117. -103	101	-101	116	...			...			...		...		...	...	...	3. 2		...	102
103 {	118. -102	101 ...	116	-101	...			...			...		...		...	...	...	2 4		...	103
104 {	119. -105	...			101	-101	116	...			...		...		...	...	...	6. 5		...	104
105 {	120. -104	...			101	116	-101	...			...		...		...	...	...	5 7		...	105
106 {		102	-103	117	...			101	-101	116	...		...		...	...	...	9. 8		...	106
107 {		103	118	-102	...			101	116	-101	...		...		...	...	...	8 10		...	107
108 {		104	-105	119	102			...		101	-101	...		...		...	...	12. 11		...	108
109 {		105	120	-104	103			...		101	116	...		...		...	...	11 13		...	109
110 {		105	-104	120	102			...		...		101		...		...	...	15. 14		...	110
111 {		104	119	-105	103			...		...		...	101		...	...	...	14 16		...	111
112 {		...			104			...		...		...		101	...	...	...	18. 17		...	112
113 {		...			105			...		...		...		101	...	...	...	17 19		...	113
114 {		...			...			...		...		...		...	...	...	101	63. 62		...	114
115 {		...			...			...		...		...		...	...	...	101	62 64		...	115
116 {	101	117 118	103	102	119 120			...		...		...		...	...	...	...	1 131		...	116

TABLE II. (Continued.)

[illegible]

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	62	101	146	147		
141 {	16	134		6	 133					12		10	131							141
142 {	18				 134	6						16		131						142
143 {	19				135		7							131						143
144 {															131			147		144
145 {															131					145
146 {	148. -147	150. -149	152. -151	154. -153	156. -155										 -146		 62	 63. 1		146
147 {	 -146	151 153	149	 -150	157 159										 148		 63 1			147
148 {	146	152 154	150	 -149	158 160										147		 1 64	 62 131		148
149 {	153. -152	 -146	 147	 -148														 2	 3	149
150 {	154. -151	146	148.	 -147													 2		 4	150
151 {	 -150	 147	 -146														 3		 2	151
152 {	 -149	 148	146														 3			152
153 {	149	147		 -146														 4	 2	153
154 {	150	148		146													 4			154
155 {	159. -158				 -146	147	 -148											 5	 6	155
156 {	160. -157				146	 148	 -147											 5	 7	156
157 {	 -156				 147	 -146												 6	 5	157
158 {	 -155				 148	 146												 6		158
159 {	155				 147		 -146											 7	 5	159
160 {	156				 148		 146											 7		160
161 {		149	 153	 -152				 -146	147	 -148								 8	 9	161
162 {		150	 154	 -151				 146	 148	 -147								 8	 10	162
163 {		 151	 -150					 147	 146									 9	 8	163
164 {		 152	 -149					 147 148	 146									 9		164

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	62	101	146	147	
165 {	...	153		149											...		10	8	165
	...									-146					...				
166 {	...	154		150				148		146					...		10		166
	...														...				
167 {	...	155			149										...		11		167
	...		159	-158							-146	147			...			-12	
168 {	...	156			150						146				...		11		168
	...		160	-157								148			...			13	
169 {	...				151										...		12		169
	...	157	-156								147	-146			...			-11	
170 {	...				152							146			...		12		170
	...	158	-155								148				...				
171 {	...	159			153						147				...		13	11	171
	...														...				
172 {	...	160		156	154						148				...		13		172
	...														...				
173 {	...														...		14		173
	...	-156	157	-160	149								-146		...			-15	
174 {	...												146		...		14		174
	...	-155	158	-159	150										...			16	
175 {	...		155		151										...		15		175
	...	159											147		...			-14	
176 {	...		156		152										...		15		176
	...	160											148		...				
177 {	...	157											147		...		16	14	177
	...			-156	153										...				
178 {	...	158											148		...		16		178
	...			-155	154										...				
179 {	...				155									-146			17		179
	...														...			-18	
180 {	...				156									146			17		180
	...														...			19	
181 {	...				157									147			18		181
	...														...			-17	
182 {	...				158									148			18		182
	...														...				
183 {	...				159									147			19	17	183
	...														...				
184 {	...				160									148			19		184
	...														...				
185 {	...														...	147.			185
	...														...	-146			
186 {	...														...	146			186
	...														...	148			

	38	59			38	59	
39 {	 1		39	60 {	 1		60
40 {	 1		40	61 {	 1		61

Table II. may be used in forming the developments required in the method employed by MM. LAPLACE and DAMOISEAU; for this purpose it is only necessary to make $t = \lambda' - \lambda_i$ instead of $nt - n_i t$

$$x = c\lambda' - \varpi \quad . \quad . \quad . \quad cnt - \varpi$$

$$z = c_i \lambda_i - \varpi_i \quad . \quad . \quad . \quad c_i n_i t - \varpi_i$$

$$\text{and } y = g\lambda' - \nu \quad . \quad . \quad . \quad gnt - \nu$$

The notation throughout is the same as that used Phil. Trans. 1830, p. 328, with the exception of the indices of the arguments.

In the elliptic movement;

$$\begin{aligned} a^5 r^{-5} = & 1 + 5e^2 \left(1 + \frac{21}{8}e^2\right) + 5e \left(1 + \frac{27}{8}e^2\right) \cos x + 10e^2 \left(1 + \frac{31}{12}e^2\right) \cos 2x \\ & + \frac{145}{8}e^3 \cos 3x + \frac{745}{48}e^4 \cos 4x \end{aligned}$$

$$a^4 r^{-4} = 1 + 3e^2 + 4e \cos x + 7e^2 \cos 2x$$

$$\begin{aligned} a^3 r^{-3} = & 1 + \frac{3}{2}e^2 \left(1 + \frac{5}{4}e^2\right) + 3e \left(1 + \frac{9}{8}e^2\right) \cos x + \frac{9}{2}e^2 \left(1 + \frac{7}{9}e^2\right) \cos 2x \\ & + \frac{53}{8}e^3 \cos 3x + \frac{77}{8}e^4 \cos 4x \end{aligned}$$

$$\begin{aligned} a^2 r^{-2} = & 1 + \frac{e^2}{2} \left(1 + \frac{3}{4}e^2\right) + 2e \left(1 + \frac{3}{8}e^2\right) \cos x + \frac{5}{2}e^2 \left(1 + \frac{2}{15}e^2\right) \cos 2x \\ & + \frac{13}{4}e^3 \cos 3x + \frac{103}{24}e^4 \cos 4x \end{aligned}$$

$$a r^{-1} = 1 + e \left(1 - \frac{e^2}{8}\right) \cos x + e^2 \left(1 - \frac{e^2}{3}\right) \cos 2x + \frac{9}{8}e^3 \cos 3x + \frac{4}{3}e^4 \cos 4x$$

$$\frac{r}{a} = 1 + \frac{e^2}{2} - e \left(1 - \frac{3e^2}{8}\right) \cos x - \frac{e^2}{2} \left(1 - \frac{2e^2}{3}\right) \cos 2x - \frac{3e^3}{8} \cos 3x - \frac{e^4}{3} \cos 4x$$

$$\frac{r^2}{a^2} = 1 + \frac{3e^2}{2} - 2e \left(1 - \frac{e^2}{8}\right) \cos x - \frac{e^2}{2} \left(1 - \frac{e^2}{3}\right) \cos 2x - \frac{e^3}{4} \cos 3x - \frac{e^4}{6} \cos 4x$$

$$\frac{r^3}{a^3} = 1 + 3e^2 \left(1 + \frac{e^2}{8}\right) - 3e \left(1 + \frac{3}{8}e^2\right) \cos x - \frac{5}{8}e^4 \cos 2x + \frac{e^3}{8} \cos 3x + \frac{e^4}{8} \cos 4x$$

$$\frac{r^4}{a^4} = 1 + 5e^2 - 4e \cos x + e^2 \cos 2x$$

$$\frac{a}{r} = r_0$$

$$+ r_1 \cos 2t$$

$$+ e r_2 \cos x$$

$$+ e r_3 \cos (2t - x)$$

$$+ e r_4 \cos (2t + x)$$

$$+ e_i r_5 \cos z$$

$$+ e_i r_6 \cos (2t - z) + \&c. \ \&c.$$

$$\begin{aligned}
\lambda &= n t \\
&+ \lambda_1 \cos 2 t \\
&+ e \lambda_2 \cos x \\
&+ e \lambda_3 \cos (2 t - x) \\
&+ e \lambda_4 \cos (2 t + x) \\
&+ e_1 \lambda_5 \cos z \text{ \&c. \&c.}
\end{aligned}$$

The quantities λ correspond to the quantities b in M. DAMOISEAU's notation.

$$\begin{aligned}
s &= \gamma s_{146} \sin y \\
&+ \gamma s_{147} \sin (2 t - y) \\
&+ \gamma s_{148} \sin (2 t + y) \\
&+ e \gamma s_{149} \sin (x - y) \text{ \&c. \&c.} \\
\gamma &= \tan i
\end{aligned}$$

$$\begin{aligned}
R &= m_i \left\{ \frac{r^1 r_i \cos (\lambda - \lambda_i)}{r_i^3} - \frac{1}{\{r^2 - 2 r^1 r_i \cos (\lambda' - \lambda_i) + r_i^2\}^{\frac{3}{2}}} \right\} \\
&= m_i \left\{ -\frac{1}{r_i} + \frac{r^2}{2 r_i^3} - \frac{3}{8} \frac{\{2 r^1 r_i \cos (\lambda' - \lambda_i) - r^2\}^2}{r_i^5} - \frac{15}{48} \frac{\{2 r^1 r_i \cos (\lambda' - \lambda_i) - r^2\}^3}{r_i^7} \right\} \\
&= m_i \left\{ -\frac{1}{r_i} + \frac{r^2}{2 r_i^3} - \frac{3}{2} \frac{r^2 r_i^2}{r_i^5} \cos (\lambda' - \lambda_i)^2 + \frac{3}{2} \frac{r^2 r^1 r_i}{r_i^5} \cos (\lambda - \lambda_i) - \frac{5}{2} \frac{r^3 r_i^3}{r_i^7} \cos (\lambda' - \lambda_i)^3 \right\} \\
&= m_i \left\{ -\frac{1}{r_i} - \frac{r^2}{4 r_i^3} \left\{ 1 + 3 \cos (2 \lambda' - 2 \lambda_i) - 2 s^2 \right\} \right. \\
&\quad \left. - \frac{r^3}{8 r_i^4} \left\{ 3 (1 - 4 s^2) \cos (\lambda' - \lambda_i) + 5 \cos (3 \lambda' - 3 \lambda_i) \right\} \right\} \\
r^1 r_i \cos (\lambda' - \lambda_i) &= r r_i \left\{ \cos^2 \frac{i}{2} \cos (\lambda - \lambda_i) + \sin^2 \frac{i}{2} \cos (\lambda + \lambda_i - 2 \nu) \right\} \\
&= * a_i \cos^2 \frac{i}{2} \left\{ \left(1 - \frac{e^2}{2} - \frac{e^4}{64} \right) \left(1 - \frac{e_i^2}{2} - \frac{e_i^4}{64} \right) \cos t - \frac{3 e}{2} \left(1 - \frac{e_i^2}{2} \right) \cos (t - x) \right. \\
&\quad + \frac{e}{2} \left(1 - \frac{3}{4} e^2 \right) \left(1 - \frac{e_i^2}{2} \right) \cos (t + x) + \frac{3}{8} e^2 (1 - e^2) \left(1 - \frac{e_i^2}{2} \right) \cos (t + 2 x) \\
&\quad + \frac{e^3}{3} \cos (t + 3 x) + \frac{125}{384} e^4 \cos (t + 4 x) + \frac{e^2}{8} \left(1 + \frac{e^2}{3} \right) \left(1 - \frac{e_i^2}{2} \right) \cos (t - 2 x) \\
&\quad + \frac{e^3}{24} \cos (t - 3 x) + \frac{3}{128} e^4 \cos (t - 4 x) - \frac{3}{2} e_i \left(1 - \frac{e^2}{2} \right) \cos (t + z) \\
&\quad + \frac{9}{4} e e_i \cos (t - x + z) - \frac{3}{4} e e_i \left(1 - \frac{3}{4} e^2 \right) \cos (t + x + z) \\
&\quad \left. - \frac{9}{16} e^2 e_i \cos (t + 2 x + z) - \frac{e^3 e_i}{2} \cos (t + 3 x + z) - \frac{3}{16} e^2 e_i \cos (t - 2 x + z) \right\}
\end{aligned}$$

* See Phil. Trans. 1830, p. 343.

$$\begin{aligned}
& -\frac{e^3 e_l \cos}{16 \sin} (t-3x+z) + \frac{e_l}{2} \left(1 - \frac{3}{4} e_l^2\right) \left(1 - \frac{e^2}{2}\right) \frac{\cos}{\sin} (t-z) \\
& -\frac{3}{4} e e_l \left(1 - \frac{3}{4} e_l^2\right) \frac{\cos}{\sin} (t-x-z) \\
& + \frac{e e_l}{4} \left(1 - \frac{3}{4} e^2\right) \left(1 - \frac{3}{4} e_l^2\right) \frac{\cos}{\sin} (t+x-z) + \frac{3}{16} e^2 e_l \frac{\cos}{\sin} (t+2x-z) \\
& + \frac{e^3 e_l \cos}{6 \sin} (t+3x-z) + \frac{e^2 e_l \cos}{16 \sin} (t-2x-z) + \frac{e^3 e_l \cos}{48 \sin} (t-3x-z) \\
& + \frac{3}{8} e_l^2 (1 - e_l^2) \left(1 - \frac{e^2}{2}\right) \frac{\cos}{\sin} (t-2z) - \frac{9}{16} e e_l^2 \frac{\cos}{\sin} (t-x-2z) \\
& + \frac{3}{16} e e_l^2 \frac{\cos}{\sin} (t+x-2z) + \frac{9}{64} e^2 e_l^2 \frac{\cos}{\sin} (t+2x-2z) \\
& + \frac{3}{64} e^2 e_l^2 \frac{\cos}{\sin} (t-2x-2z) + \frac{e_l^3 \cos}{3 \sin} (t-3z) - \frac{e e_l^3 \cos}{2 \sin} (t-x-3z) \\
& + \frac{e e_l^3 \cos}{6 \sin} (t+x-3z) + \frac{125}{384} e_l^4 \frac{\cos}{\sin} (t-4z) \\
& + \frac{e_l^2}{8} \left(1 + \frac{e_l^2}{3}\right) \left(1 - \frac{e^2}{2}\right) \frac{\cos}{\sin} (t+2z) - \frac{3}{16} e e_l^2 \frac{\cos}{\sin} (t-x+2z) \\
& + \frac{e e_l^2 \cos}{16 \sin} (t+x+2z) + \frac{3}{64} e^2 e_l^2 \frac{\cos}{\sin} (t+2x+2z) + \frac{e^2 e_l^2 \cos}{64 \sin} (t-2x+2z) \\
& + \frac{e_l^3 \cos}{24 \sin} (t+3z) - \frac{e e_l^3 \cos}{16 \sin} (t-x+3z) + \frac{e e_l^3 \cos}{48 \sin} (t+x+3z) \\
& + \frac{3}{128} e_l^4 \frac{\cos}{\sin} (t+4z) \} \\
& + a a_l \sin^2 \frac{t}{2} \left\{ \left(1 - \frac{e^2 + e_l^2}{2}\right) \frac{\cos}{\sin} (t-2y) - \frac{3}{2} e \frac{\cos}{\sin} (t+x-2y) + \frac{e}{2} \frac{\cos}{\sin} (t-x-2y) \right. \\
& + \frac{3}{8} e^2 \frac{\cos}{\sin} (t-2x-2y) + \frac{e^2 \cos}{8 \sin} (t+2x-2y) - \frac{3}{2} e_l \frac{\cos}{\sin} (t+z-2y) \\
& + \frac{9}{4} e e_l \frac{\cos}{\sin} (t+x+z-2y) - \frac{3}{4} e e_l \frac{\cos}{\sin} (t-x+z-2y) \\
& \left. + \frac{e_l \cos}{2 \sin} (t-z-2y) - \frac{3}{4} e e_l \frac{\cos}{\sin} (t+x-z-2y) + \frac{e e_l \cos}{4 \sin} (t-x-z-2y) \right\} \\
& r^{\frac{1}{2}} r_l^2 \cos (\lambda' - \lambda_l)^2 \\
& = a^2 a_l^2 \cos^4 \frac{t}{2} \left\{ \frac{1}{2} + \left\{ -\frac{1}{2} + \frac{9}{8} + \frac{1}{8} \right\} (e^2 + e_l^2) + \left\{ \frac{1}{2} - \frac{9}{8} - \frac{1}{8} - \frac{9}{8} + \frac{81}{32} + \frac{9}{32} \right. \right. \\
& \quad \left. \left. - \frac{1}{8} + \frac{9}{32} + \frac{1}{32} \right\} e^2 e_l^2 + \left\{ \frac{7}{64} - \frac{3}{16} + \frac{9}{128} + \frac{1}{128} \right\} (e^4 + e_l^4) \right\}
\end{aligned}$$

$$\begin{aligned}
& + \left\{ \frac{1}{2} + \left\{ -\frac{1}{2} - \frac{3}{4} \right\} (e^2 + e_i^2) + \left\{ \frac{1}{2} + \frac{3}{4} + \frac{3}{4} + \frac{9}{16} + \frac{9}{16} \right\} e^2 e_i^2 \right. \\
& \quad \left. + \left\{ \frac{7}{64} + \frac{9}{16} + \frac{3}{64} \right\} (e^4 + e_i^4) \right\} \cos 2t \\
& \hspace{15em} [1] \\
& + \left\{ -\frac{3}{2} + \frac{1}{2} + \left\{ \frac{3}{4} - \frac{5}{8} - \frac{3}{16} + \frac{3}{16} \right\} e^2 \right. \\
& \quad \left. + \left\{ \frac{3}{2} - \frac{1}{2} - \frac{27}{8} + \frac{9}{8} - \frac{3}{8} + \frac{1}{8} \right\} e_i^2 \right\} e \cos x \\
& \hspace{15em} [2] [5]^* \\
& + \left\{ -\frac{3}{2} + \left\{ \frac{3}{4} + \frac{1}{16} \right\} e^2 + \left\{ \frac{3}{2} + \frac{9}{8} + \frac{9}{8} \right\} e_i^2 \right\} e \cos (2t - x) \\
& \hspace{15em} [3] [7] \\
& + \left\{ -\frac{1}{2} + \left\{ \frac{5}{8} - \frac{9}{16} \right\} e^2 + \left\{ -\frac{1}{2} - \frac{3}{8} - \frac{3}{8} \right\} e_i^2 \right\} e \cos (2t + x) \\
& \hspace{15em} [4] [6] \\
& + \left\{ \frac{3}{8} + \frac{1}{8} - \frac{3}{4} + \left\{ -\frac{9}{16} - \frac{1}{48} + \frac{9}{16} - \frac{1}{16} + \frac{1}{6} \right\} e^2 \right. \\
& \quad \left. + \left\{ -\frac{3}{8} - \frac{1}{8} + \frac{3}{4} + \frac{27}{32} + \frac{9}{32} - \frac{27}{16} + \frac{3}{32} + \frac{1}{32} - \frac{3}{16} \right\} e_i^2 \right\} e^2 \cos 2x \\
& \hspace{15em} [8] [17] \\
& + \left\{ \frac{9}{8} + \frac{1}{8} + \left\{ -\frac{1}{48} + \frac{1}{48} \right\} e^2 \right. \\
& \quad \left. + \left\{ -\frac{9}{8} - \frac{1}{8} - \frac{3}{32} - \frac{27}{16} - \frac{3}{32} \right\} e_i^2 \right\} e^2 \cos (2t - 2x) \\
& \hspace{15em} [9] [19] \\
& + \left\{ \frac{1}{8} + \frac{3}{8} + \left\{ -\frac{3}{16} - \frac{9}{16} - \frac{1}{2} \right\} e^2 \right. \\
& \quad \left. + \left\{ -\frac{1}{8} - \frac{3}{8} - \frac{9}{32} - \frac{3}{16} - \frac{9}{32} \right\} e_i^2 \right\} e^2 \cos (2t + 2x) \\
& \hspace{15em} [10] [18] \\
& + \left\{ -\frac{3}{4} - \frac{3}{4} + \frac{9}{4} + \frac{1}{4} + \left\{ \frac{15}{16} + \frac{3}{8} - \frac{9}{8} - \frac{3}{32} - \frac{9}{32} - \frac{5}{16} \right. \right. \\
& \quad \left. \left. + \frac{3}{32} + \frac{9}{32} \right\} (e^2 + e_i^2) \right\} e e_i \cos (x + z) \\
& \hspace{15em} [11]
\end{aligned}$$

* The coefficient of argument 5 being the same, e and e_i changing places, that coefficient is not written down, in order to avoid useless repetition.

$$\begin{aligned}
& + \left\{ -\frac{3}{4} - \frac{3}{4} + \left\{ \frac{3}{8} + \frac{3}{8} + \frac{1}{32} + \frac{1}{32} \right\} e^2 \right. \\
& \quad \left. + \left\{ \frac{15}{16} + \frac{15}{16} + \frac{27}{32} + \frac{27}{32} \right\} e_i^2 \right\} e e_i \cos (2t - x - z) \\
& \qquad \qquad \qquad [12] [13] \\
& + \left\{ \frac{9}{4} + \frac{1}{4} - \frac{3}{4} - \frac{3}{4} + \left\{ -\frac{9}{8} - \frac{5}{16} + \frac{9}{32} + \frac{3}{8} + \frac{15}{16} \right. \right. \\
& \quad \left. \left. + \frac{3}{32} - \frac{9}{32} - \frac{3}{32} \right\} (e^2 + e_i^2) \right\} e e_i \cos (x - z) \\
& \qquad \qquad \qquad [14] \\
& + \left\{ \frac{9}{4} + \frac{9}{4} + \left\{ -\frac{9}{8} - \frac{9}{8} - \frac{3}{32} - \frac{3}{32} \right\} (e^2 + e_i^2) \right\} e e_i \cos (2t - x + z) \\
& \qquad \qquad \qquad [15] \\
& + \left\{ \frac{1}{4} + \frac{1}{4} + \left\{ -\frac{5}{16} - \frac{9}{32} - \frac{5}{16} - \frac{9}{32} \right\} (e_i + e_i^2) \right\} \cos (2t + x - z) \\
& \qquad \qquad \qquad [16] \\
& + \left\{ \frac{1}{3} + \frac{1}{24} - \frac{9}{16} + \frac{1}{16} \right\} e \cos 3x \\
& \qquad \qquad \qquad [20] [35] \\
& + \left\{ \frac{1}{24} - \frac{3}{16} \right\} e^3 \cos (2t - 3x) \\
& \qquad \qquad \qquad [21] [37] \\
& + \left\{ \frac{1}{3} + \frac{3}{16} \right\} e^3 \cos (2t + 3x) \\
& \qquad \qquad \qquad [22] [36] \\
& + \left\{ -\frac{9}{16} + \frac{1}{16} + \frac{9}{8} - \frac{3}{8} + \frac{3}{16} - \frac{3}{16} \right\} e^2 e_i \cos (2x + z) \\
& \qquad \qquad \qquad [23] [29] \\
& + \left\{ \frac{1}{16} + \frac{9}{8} + \frac{1}{16} \right\} e^2 e_i \cos (2t - 2x - z) \\
& \qquad \qquad \qquad [24] [31] \\
& + \left\{ -\frac{9}{16} - \frac{3}{8} - \frac{9}{16} \right\} e^2 e_i \cos (2t + 2x + z) \\
& \qquad \qquad \qquad [25] [30] \\
& + \left\{ -\frac{3}{16} + \frac{3}{16} - \frac{3}{8} + \frac{9}{8} - \frac{9}{16} + \frac{1}{16} \right\} e^2 e_i \cos (2x - z) \\
& \qquad \qquad \qquad [26] [32] \\
& + \left\{ -\frac{3}{16} - \frac{27}{8} - \frac{3}{16} \right\} e^2 e_i \cos (2t - 2x + z) \\
& \qquad \qquad \qquad [27] [33] \\
& + \left\{ \frac{3}{16} + \frac{1}{8} + \frac{3}{16} \right\} e^2 e_i \cos (2t + 2x - z) \\
& \qquad \qquad \qquad [28] [34] \\
& + \left\{ \frac{125}{384} + \frac{3}{128} - \frac{1}{2} + \frac{1}{48} + \frac{3}{64} \right\} e^4 \cos 4x \\
& \qquad \qquad \qquad [38] [59]
\end{aligned}$$

$$+ \left\{ \frac{1}{128} + \frac{3}{128} - \frac{1}{16} \right\} e^4 \cos(2t - 4x)$$

[39] [61]

$$+ \left\{ \frac{9}{128} + \frac{125}{384} + \frac{1}{6} \right\} e^4 \cos(2t + 4x)$$

[40] [60]

$$+ \left\{ -\frac{1}{2} + \frac{1}{48} + \frac{27}{32} + \frac{1}{32} - \frac{9}{32} + \frac{1}{6} - \frac{3}{32} - \frac{1}{16} \right\} e^3 \cos(3x + z)$$

[41] [53]

$$+ \left\{ \frac{1}{48} - \frac{3}{32} - \frac{3}{32} + \frac{1}{48} \right\} e^3 e_i \cos(2t - 3x - z)$$

[42] [55]

$$+ \left\{ -\frac{1}{2} - \frac{9}{32} - \frac{9}{32} - \frac{1}{2} \right\} e^3 e_i \cos(2t + 3x + z)$$

[43] [54]

$$+ \left\{ -\frac{1}{16} + \frac{1}{6} - \frac{9}{32} - \frac{3}{32} + \frac{27}{32} - \frac{1}{2} + \frac{1}{32} + \frac{1}{48} \right\} e^3 e_i \cos(3x - z)$$

[44] [56]

$$+ \left\{ -\frac{1}{16} + \frac{9}{32} + \frac{9}{32} - \frac{1}{16} \right\} e^3 e_i \cos(2t - 3x + z)$$

[45] [57]

$$+ \left\{ \frac{1}{6} + \frac{3}{32} + \frac{3}{32} + \frac{1}{6} \right\} e^3 e_i \cos(2t + 3x - z)$$

[46] [58]

$$+ \left\{ \frac{3}{64} + \frac{3}{64} - \frac{3}{32} - \frac{9}{32} + \frac{9}{64} + \frac{1}{64} - \frac{3}{32} + \frac{9}{16} - \frac{9}{32} \right\} e^2 e_i^2 \cos(2x + 2z)$$

[47]

$$+ \left\{ \frac{9}{32} + \frac{3}{64} + \frac{27}{32} + \frac{3}{64} + \frac{1}{32} \right\} e^2 e_i^2 \cos(2t - 2x + 2z)$$

[48]

$$+ \left\{ \frac{9}{32} + \frac{3}{64} + \frac{1}{32} + \frac{3}{64} + \frac{27}{32} \right\} e^2 e_i^2 \cos(2t + 2x + 2z)$$

[49]

$$+ \left\{ \frac{9}{64} + \frac{1}{64} - \frac{9}{32} - \frac{3}{32} + \frac{3}{64} + \frac{3}{64} - \frac{9}{32} + \frac{9}{16} - \frac{3}{32} \right\} e^2 e_i^2 \cos(2x - 2z)$$

[50]

$$+ \left\{ \frac{81}{32} + \frac{1}{64} + \frac{9}{32} + \frac{1}{64} + \frac{9}{32} \right\} e^2 e_i^2 \cos(2t - 2x + 2z)$$

[51]

$$+ \left\{ \frac{1}{32} + \frac{9}{64} + \frac{3}{32} + \frac{9}{64} + \frac{3}{32} \right\} e^2 e_i^2 \cos(2t + 2x - 2z)$$

[52]

$$+ a^2 a_i^2 \sin^2 \frac{t}{2} \cos^2 \frac{t}{2} \left\{ \left\{ 1 + \left\{ -1 - \frac{3}{4} - \frac{3}{4} \right\} e^2 + \left\{ -1 + \frac{9}{4} + \frac{1}{4} \right\} e_i^2 \right\} \cos 2y \right.$$

[62]

$$+ \left\{ 1 + \left\{ -1 + \frac{9}{4} + \frac{1}{4} \right\} e^2 + \left\{ -1 - \frac{3}{4} - \frac{3}{4} \right\} e_i^2 \right\} \cos(2t - 2y)$$

[63]

$$+ \left\{ -\frac{3}{2} - \frac{3}{2} \right\} e \cos (x - 2y) + \left\{ \frac{1}{2} + \frac{1}{2} \right\} e \cos (x + 2y)$$

[65]
[66]

$$+ \left\{ \frac{1}{2} - \frac{3}{2} \right\} e \cos (2t - x - 2y)$$

[67]

$$+ \left\{ -\frac{3}{2} + \frac{1}{2} \right\} e \cos (2t + x - 2y) + \left\{ -\frac{3}{2} + \frac{1}{2} \right\} e_1 \cos (z - 2y)$$

[69]
[71]

$$+ \left\{ \frac{1}{2} - \frac{3}{2} \right\} e_1 \cos (z + 2y) + \left\{ \frac{1}{2} + \frac{1}{2} \right\} e_1 \cos (2t - z - 2y)$$

[72]
[73]

$$+ \left\{ -\frac{3}{2} - \frac{3}{2} \right\} e_1 \cos (2t + z - 2y) + \left\{ \frac{1}{8} + \frac{9}{4} + \frac{1}{8} \right\} e^2 \cos (2x - 2y)$$

[75]
[77]

$$+ \left\{ \frac{3}{8} + \frac{1}{4} + \frac{3}{8} \right\} e^2 \cos (2x + 2y)$$

[78]

$$+ \left\{ \frac{3}{8} - \frac{3}{4} + \frac{1}{8} \right\} e^2 \cos (2t - 2x - 2y) + \left\{ \frac{1}{8} - \frac{3}{4} + \frac{3}{8} \right\} e^2 \cos (2t + 2x - 2y)$$

[79]
[81]

$$+ \left\{ \frac{9}{4} + \frac{9}{4} - \frac{3}{4} - \frac{3}{4} \right\} e e_1 \cos (x + z - 2y)$$

[83]

$$+ \left\{ \frac{1}{4} + \frac{1}{4} - \frac{3}{4} - \frac{3}{4} \right\} e e_1 \cos (x + z + 2y)$$

[84]

$$+ \left\{ \frac{1}{4} - \frac{3}{4} + \frac{1}{4} - \frac{3}{4} \right\} e e_1 \cos (2t - x - z - 2y)$$

[85]

$$+ \left\{ \frac{9}{4} - \frac{3}{4} + \frac{9}{4} - \frac{3}{4} \right\} e e_1 \cos (2t + x + z + 2y)$$

[87]

$$+ \left\{ -\frac{3}{4} - \frac{3}{4} + \frac{9}{4} + \frac{9}{4} \right\} e e_1 \cos (x - z - 2y)$$

[89]

$$+ \left\{ -\frac{3}{4} - \frac{3}{4} + \frac{1}{4} + \frac{1}{4} \right\} e e_1 \cos (x - z + 2y)$$

[90]

$$+ \left\{ -\frac{3}{4} + \frac{9}{4} - \frac{3}{4} + \frac{9}{4} \right\} e e_1 \cos (2t - x + z - 2y)$$

[91]

$$+ \left\{ -\frac{3}{4} + \frac{1}{4} - \frac{3}{4} + \frac{1}{4} \right\} e e_1 \cos (2t + x - z - 2y)$$

[93]

$$+ \left\{ -\frac{3}{4} + \frac{3}{8} \right\} e_i^2 \cos(2z - 2y) + \left\{ -\frac{3}{4} + \frac{1}{8} \right\} e_i^2 \cos(2z + 2y) \quad [95] \quad [96]$$

$$+ \left\{ \frac{1}{4} + \frac{3}{8} \right\} e_i^2 \cos(2t - 2z - 2y) + \left\{ \frac{9}{4} + \frac{1}{8} \right\} e_i^2 \cos(2t + 2z - 2y) \quad [97] \quad [99]$$

$$+ a^2 a_i^2 \sin^4 \frac{t}{2} \left\{ \frac{1}{2} + \frac{1}{2} \cos(2t - 2y) \right\} \quad [63]$$

$$r^2 r^2 \cos(\lambda' - \lambda)^2$$

$$= a^2 a_i^2 \cos^4 \frac{t}{2} \left\{ \frac{1}{2} + \frac{3}{4} (e^2 + e_i^2) + \frac{9}{8} e^2 e_i^2 + \left\{ \frac{1}{2} - \frac{5}{4} (e^2 + e_i^2) \right. \right. \\ + \frac{23}{32} (e^4 + e_i^4) + \frac{25}{8} e^2 e_i^2 \left. \right\} \cos 2t + \left\{ -1 + \frac{e^2}{8} - \frac{3}{2} e_i^2 \right\} e \cos x \quad [1] \quad [2] \quad [5] \\ + \left\{ -\frac{3}{2} + \frac{13}{16} e^2 + \frac{15}{4} e_i^2 \right\} e \cos(2t - x) + \left\{ \frac{1}{2} - \frac{19}{16} e^2 - \frac{5}{4} e_i^2 \right\} e \cos(2t + x) \quad [3] \quad [7] \quad [4] \quad [6] \\ + \left\{ -\frac{1}{4} + \frac{1}{12} e^2 - \frac{3}{8} e_i^2 \right\} e^2 \cos 2x \quad [8] \quad [17] \\ + \left\{ \frac{5}{4} - \frac{25}{8} e_i^2 \right\} e^2 \cos(2t - 2x) + \left\{ \frac{1}{2} - \frac{5}{4} e^2 - \frac{5}{4} e_i^2 \right\} e \cos(2t + 2x) \quad [9] \quad [19] \quad [10] \quad [18] \\ + \left\{ 1 - \frac{1}{8} (e^2 + e_i^2) \right\} e e_i \cos(x + z) \quad [11] \\ + \left\{ -\frac{3}{2} + \frac{13}{16} e^2 + \frac{57}{16} e_i^2 \right\} e e_i \cos(2t - x - z) \quad [12] \quad [13] \\ + \left\{ 1 - \frac{(e^2 + e_i^2)}{8} \right\} e e_i \cos(x - z) \quad [14] \\ + \left\{ \frac{9}{2} - \frac{39}{16} (e^2 + e_i^2) \right\} e e_i \cos(2t - x + z) \quad [15] \\ + \left\{ \frac{1}{2} - \frac{19}{16} (e^2 + e_i^2) \right\} e e_i \cos(2t + x - z) - \frac{e^3}{8} \cos 3x \quad [16] \quad [20] \quad [35] \\ - \frac{7e^3}{48} \cos(2t - 3x) + \frac{25e^3}{48} \cos(2t + 3x) + \frac{e^2 e_i}{4} \cos(2x + z) \quad [21] \quad [37] \quad [22] \quad [36] \quad [23] \quad [29]$$

$$\begin{aligned}
& + \frac{5}{4} e^2 e_i \cos(2t - 2x - z) - \frac{25}{16} e^2 e_i \cos(2t + 2x + z) + \frac{e^2 e_i}{4} \cos(2x - z) \\
& \quad [24] [31] \qquad [25] [30] \qquad [26] [32] \\
& - \frac{15}{4} e^2 e_i \cos(2t - 2x + z) + \frac{e^2 e_i}{2} \cos(2t + 2x - z) \\
& \quad [27] [33] \qquad [28] [34] \\
& - \frac{e^4}{12} \cos 4x - \frac{e^4}{32} \cos(2t - 4x) + \frac{9}{16} e^4 \cos(2t + 4x) + \frac{e^3 e_i}{8} \cos(3x + z) \\
& \quad [38] [59] \qquad [39] [61] \qquad [40] [60] \qquad [41] [53] \\
& - \frac{7}{48} e^3 e_i \cos(2t - 3x - z) - \frac{25}{16} e^2 e_i \cos(2t + 3x + z) \\
& \quad [42] [55] \qquad [43] [54] \\
& + \frac{e^3 e_i}{8} \cos(3x - z) + \frac{7}{16} e^3 e_i \cos(2t - 3x + z) + \frac{25}{48} e^3 e_i \cos(2t + 3x - z) \\
& \quad [44] [56] \qquad [45] [57] \qquad [46] [58] \\
& + \frac{e^2 e_i}{16} \cos(2x + 2z) + \frac{5}{4} e^2 e_i^2 \cos(2t - 2x - 2z) \\
& \quad [47] \qquad [48] \\
& + \frac{5}{4} e^2 e_i \cos(2t + 2x + 2z) + \frac{e^2 e_i^2}{16} \cos(2x - 2z) \\
& \quad [49] \qquad [50] \\
& + \frac{25}{8} e^2 e_i^2 \cos(2t - 2x + 2z) + \frac{e^2 e_i^2}{2} \cos(2t + 2x - 2z) \\
& \quad [51] \qquad [52] \\
& + a^3 a_i^2 \cos^2 \frac{i}{2} \sin^2 \frac{i}{2} \left\{ \left\{ 1 - \frac{5}{2} e^2 + \frac{3}{2} e_i^2 \right\} \cos 2y + \left\{ 1 + \frac{3}{2} e^2 - \frac{5}{2} e_i^2 \right\} \cos(2t - 2y) \right. \\
& \quad [62] \qquad [63] \\
& - 3e \cos(x - 2y) + e \cos(x + 2y) - e \cos(2t - x - 2y) \\
& \quad [65] \qquad [66] \qquad [67] \\
& - e \cos(2t + x - 2y) - e_i \cos(z - 2y) - e_i \cos(z + 2y) \\
& \quad [69] \qquad [71] \qquad [72] \\
& + e_i \cos(2t - z - 2y) - 3e_i \cos(2t + z - 2y) \\
& \quad [73] \qquad [75] \\
& + \frac{5}{2} e^2 \cos(2x - 2y) + e^2 \cos(2x + 2y) \\
& \quad [77] \qquad [78] \\
& - \frac{e^2}{4} \cos(2t - 2x - 2y) - \frac{e^2}{4} \cos(2t + 2x - 2y) \\
& \quad [79] \qquad [81]
\end{aligned}$$

$$+ 3 e e_i \cos (x + z - 2 y) - e e_i \cos (x + z + 2 y) \\ [83] \qquad [84]$$

$$- e e_i \cos (2 t - x - z - 2 y) + 3 e e_i \cos (2 t + x + z - 2 y) \\ [85] \qquad [87]$$

$$+ 3 e e_i \cos (x - z - 2 y) - e e_i \cos (x - z + 2 y) \\ [89] \qquad [90]$$

$$+ 3 e e_i \cos (2 t - x + z - 2 y) - e e_i \cos (2 t + x - z - 2 y) \\ [91] \qquad [93]$$

$$- \frac{3}{8} e_i^2 \cos (2 z - 2 y) - \frac{5}{8} e_i^2 \cos (2 z + 2 y) \\ [95] \qquad [96]$$

$$+ \frac{5 e_i^2}{8} \cos (2 t - 2 z - 2 y) + \frac{19}{8} e_i^2 \cos (2 t + 2 z - 2 y) \} \\ [97] \qquad [99]$$

$$+ a^2 a_i^2 \sin^4 \frac{t}{2} \left\{ \frac{1}{2} + \frac{1}{2} \cos (2 t - 2 y) \right\} \\ [63]$$

$$\frac{r^2}{2 r_i^3} = \frac{a^2}{a_i^3} \left\{ \frac{1}{2} + \frac{3}{4} e^2 + \frac{3}{4} e_i^2 + \frac{9}{8} e^2 e_i^2 + \frac{15}{16} e_i^4 - e \left\{ 1 - \frac{e^2}{8} + \frac{3}{2} e_i^2 \right\} \cos x \right. \\ [2]$$

$$\left. + \frac{3}{2} e_i \left\{ 1 + \frac{3}{2} e^2 + \frac{9}{8} e_i^2 \right\} \cos z - \frac{e^2}{4} \left\{ 1 - \frac{e^2}{3} + \frac{3 e_i^2}{2} \right\} \cos 2 x \right. \\ [5] \qquad [8]$$

$$- \frac{3}{2} e e_i \left\{ 1 - \frac{e^2}{8} + \frac{9}{8} e_i^2 \right\} \cos (x + z) \\ [11]$$

$$- \frac{3}{2} e e_i \left\{ 1 - \frac{e^2}{8} + \frac{9}{8} e_i^2 \right\} \cos (x - z) \\ [14]$$

$$+ \frac{9}{4} e_i^3 \left\{ 1 + \frac{7}{9} e_i^2 + \frac{3}{2} e^2 \right\} \cos 2 z - \frac{e^3}{8} \cos 3 x - \frac{3}{8} e^2 e_i \cos (2 x + z) \\ [17] \qquad [20] \qquad [23]$$

$$- \frac{3}{8} e^2 e_i \cos (2 x - z) - \frac{9}{4} e e_i^3 \cos (x + 2 z) - \frac{9}{4} e e_i^3 \cos (x - 2 z) \\ [26] \qquad [29] \qquad [32]$$

$$+ \frac{53}{16} e_i^3 \cos 3 z - \frac{e^4}{12} \cos 4 x - \frac{3 e^3 e_i}{16} \cos (3 x + z) \\ [35] \qquad [38] \qquad [41]$$

$$-\frac{3}{16} e^3 e_i \cos (3x - z) - \frac{9}{16} e^2 e_i^2 \cos (2x + 2z) - \frac{9}{16} e^2 e_i \cos (2x - 2z)$$

[44]
[47]
[50]

$$-\frac{53}{16} e e_i^3 \cos (x + 3z) - \frac{53}{16} e e_i^3 \cos (x - 3z) + \frac{77}{16} e_i^4 \cos 4z$$

[53]
[56]
[59]

Terms in R multiplied by $-\frac{3}{2} \cos^4 \frac{t}{2} \frac{a^2}{a_i^3}$

$$\begin{aligned}
 = & * \left\{ \frac{1}{2} + \frac{3}{4} (e^2 + e_i^2) + \frac{9}{8} e^2 e_i^2 \right\} \left\{ 1 + 5 e_i^2 + \frac{105}{8} e_i^4 \right\} \\
 & \quad \quad \quad [0] \\
 & + \left\{ -1 + \frac{e_i^2}{8} - \frac{3}{2} e \right\} \left\{ \frac{5}{2} e_i^2 + \frac{135}{16} e_i^4 \right\} - \frac{5}{4} e_i^4 \\
 & + \left\{ \left\{ \frac{1}{2} - \frac{5}{4} (e^2 + e_i^2) + \frac{23}{32} (e^4 + e_i^4) + \frac{25}{8} e^2 e_i^2 \right\} \left\{ 1 + 5 e_i^2 + \frac{105}{8} e_i^4 \right\} \right. \\
 & \quad + \left\{ \frac{1}{2} - \frac{19}{16} e_i^2 - \frac{5}{4} e^2 - \frac{3}{2} + \frac{13}{16} e_i^2 + \frac{15}{4} e^2 \right\} \left\{ \frac{5}{2} e_i^2 + \frac{135}{16} e_i^4 \right\} \\
 & \quad \left. + \left\{ \frac{5}{2} + \frac{25}{4} \right\} e_i^4 \right\} \cos 2t \\
 & \quad \quad \quad [1] \\
 & + \left\{ -1 + \frac{e^2}{8} - \frac{3}{2} e_i^2 - 5 e_i^2 + \frac{5}{2} e_i^2 + \frac{5}{2} e_i^2 \right\} e \cos x \\
 & \quad \quad \quad [2] \\
 & + \left\{ -\frac{3}{2} + \frac{13}{16} e^2 + \frac{15}{4} e_i^2 - \frac{15}{2} e_i^2 - \frac{15}{4} e_i^2 + \frac{45}{4} e_i^2 \right\} e \cos (2t - x) \\
 & \quad \quad \quad [3] \\
 & + \left\{ \frac{1}{2} - \frac{19}{16} e^2 - \frac{5}{4} e_i^2 + \frac{5}{2} e_i^2 + \frac{5}{4} e_i^2 - \frac{15}{4} e_i^2 \right\} e \cos (2t + x) \\
 & \quad \quad \quad [4] \\
 & + \left\{ -1 + \frac{e_i^2}{8} - \frac{3}{2} e^2 - 5 e_i^2 + \frac{5}{2} + \frac{15}{4} e^2 + \frac{15}{4} e_i^2 + \frac{135}{16} e_i^2 - \frac{5}{8} e_i^2 - 5 e_i^2 \right\} e_i \cos z \\
 & \quad \quad \quad [5] \\
 & + \left\{ \frac{1}{2} - \frac{19}{16} e_i^2 - \frac{5}{4} e^2 + \frac{5}{2} e_i^2 + \frac{5}{4} e_i^2 + \frac{5}{4} - \frac{25}{8} e^2 - \frac{25}{8} e_i^2 \right. \\
 & \quad \quad \left. + \frac{135}{32} e_i^2 - \frac{15}{2} e_i^2 \right\} e_i \cos (2t - z) \\
 & \quad \quad \quad [6]
 \end{aligned}$$

* This multiplication of $r^2 r_i^2 \cos (\lambda' - \lambda_i)^2$ by r_i^5 may be effected at once by means of Table II.

$$+ \left\{ -\frac{3}{2} + \frac{13}{16}e_i^2 + \frac{15}{4}e^2 - \frac{15}{2}e_i^2 + \frac{5}{4} - \frac{25}{8}e^2 - \frac{25}{8}e_i^2 + \frac{135}{32}e_i^2 + \frac{25}{8}e_i^2 + \frac{5}{2}e_i^2 \right\} e_i \cos(2t+z)$$

[7]

$$+ \left\{ -\frac{1}{4} + \frac{e^2}{12} - \frac{3}{8}e_i^2 - \frac{5}{4}e_i^2 + \frac{5}{8}e_i^2 + \frac{5}{8}e_i^2 \right\} e^2 \cos 2x$$

[8]

$$+ \left\{ \frac{5}{4} - \frac{25}{8}e_i^2 + \frac{25}{4}e_i^2 + \frac{25}{8}e_i^2 - \frac{75}{8}e_i^2 \right\} e^2 \cos(2t-2x)$$

[9]

$$+ \left\{ \frac{1}{2} - \frac{5}{4}e^2 - \frac{5}{4}e_i^2 + \frac{5}{2}e_i^2 + \frac{5}{4}e_i^2 - \frac{15}{4}e_i^2 \right\} e^2 \cos(2t+2x)$$

[10]

$$+ \left\{ 1 - \frac{e^2}{8} - \frac{e_i^2}{8} + 5e_i^2 - \frac{5}{2} + \frac{5}{16}e^2 - \frac{15}{4}e_i^2 - \frac{135}{16}e_i^2 + \frac{5}{8}e_i^2 + 5e_i^2 \right\} e e_i \cos(x+z)$$

[11]

$$+ \left\{ -\frac{3}{2} + \frac{13}{16}e^2 + \frac{57}{16}e_i^2 - \frac{15}{2}e_i^2 - \frac{15}{4}e_i^2 - \frac{15}{4} + \frac{65}{32}e^2 + \frac{75}{8}e_i^2 - \frac{405}{32}e_i^2 + \frac{45}{2}e_i^2 \right\} e e_i \cos(2t-x-z) + \left\{ -\frac{3}{2} + \frac{13}{16}e_i^2 + \frac{57}{16}e^2 - \frac{15}{2}e_i^2 + \frac{5}{4} \right.$$

[12]

$$\left. - \frac{95}{32}e^2 - \frac{25}{8}e_i^2 + \frac{135}{32}e_i^2 + \frac{25}{8}e_i^2 + \frac{5}{2}e_i^2 \right\} e e_i \cos(2t+x+z)$$

[13]

$$+ \left\{ 1 - \frac{e^2}{8} - \frac{e_i^2}{8} + 5e_i^2 + \frac{5}{8}e_i^2 - \frac{5}{2} + \frac{5}{16}e^2 - \frac{15}{4}e_i^2 - \frac{135}{16}e_i^2 + 5e_i^2 \right\} e e_i \cos(x-z)$$

[14]

$$+ \left\{ \frac{9}{2} - \frac{39}{16}e^2 - \frac{39}{16}e_i^2 + \frac{45}{2}e_i^2 - \frac{15}{4} + \frac{65}{32}e^2 + \frac{75}{8}e_i^2 - \frac{405}{32}e_i^2 - \frac{75}{8}e_i^2 - \frac{15}{2}e_i^2 \right\} e e_i \cos(2t-x+z)$$

[15]

$$+ \left\{ \frac{1}{2} - \frac{19}{16}e^2 - \frac{19}{16}e_i^2 + \frac{5}{2}e_i^2 + \frac{5}{4}e_i^2 + \frac{5}{4} - \frac{95}{32}e^2 - \frac{25}{8}e_i^2 + \frac{135}{32}e_i^2 - \frac{15}{2}e_i^2 \right\} e e_i \cos(2t+x-z)$$

[16]

$$+ \left\{ -\frac{1}{4} + \frac{e_i^2}{12} - \frac{3}{8}e^2 - \frac{5}{4}e_i^2 - \frac{5}{2} + \frac{5}{16}e_i^2 - \frac{15}{4}e^2 - \frac{135}{16}e_i^2 - \frac{5}{16}e_i^2 + 5 \right.$$

[17]

$$\left. + \frac{15}{2}e^2 + \frac{15}{2}e_i^2 + \frac{155}{12}e_i^2 - \frac{145}{16}e_i^2 \right\} e_i^2 \cos 2z$$

$$+ \left\{ \frac{1}{2} - \frac{5}{4}e_i^2 - \frac{5}{4}e^2 + \frac{5}{2}e_i^2 + \frac{125}{96}e_i^2 + \frac{5}{4} - \frac{95}{32}e_i^2 - \frac{25}{8}e^2 + \frac{135}{32}e_i^2 + \frac{5}{2} - \frac{25}{4}e^2 - \frac{25}{4}e_i^2 \right.$$

$$\left. + \frac{155}{24}e_i^2 - \frac{435}{32}e_i^2 \right\} e_i^2 \cos(2t-2z)$$

[18]

$$\begin{aligned}
& + \left\{ \frac{5}{4} - \frac{25}{8} e^2 + \frac{25}{4} e_i^2 - \frac{15}{4} + \frac{65}{32} e_i^2 + \frac{75}{8} e^2 - \frac{405}{32} e_i^2 - \frac{35}{96} e_i^2 + \frac{5}{2} - \frac{25}{4} e^2 - \frac{25}{4} e_i^2 \right. \\
& \quad \left. + \frac{155}{24} e_i^2 + \frac{145}{32} e_i^2 \right\} e_i^2 \cos (2t + 2z) \\
& \hspace{10cm} [19]
\end{aligned}$$

$$\begin{aligned}
& - \frac{e^3}{8} \cos 3x - \frac{7}{48} e^3 \cos (2t - 3x) + \frac{25}{48} e^3 \cos (2t + 3x) + \left\{ \frac{1}{4} - \frac{5}{8} \right\} e^2 e_i \cos (2x + z) \\
& \hspace{10cm} [20] \hspace{10cm} [21] \hspace{10cm} [22] \hspace{10cm} [23]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ \frac{5}{4} + \frac{25}{8} \right\} e^2 e_i \cos (2t - 2x - z) - \left\{ -\frac{3}{2} + \frac{5}{4} \right\} e^2 e_i \cos (2t + 2x + z) \\
& \hspace{10cm} [24] \hspace{10cm} [25]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ \frac{1}{4} - \frac{5}{8} \right\} e^2 e_i \cos (2x - z) + \left\{ -\frac{15}{4} + \frac{25}{8} \right\} e^2 e_i \cos (2t - 2x + z) \\
& \hspace{10cm} [26] \hspace{10cm} [27]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ \frac{1}{2} + \frac{5}{4} \right\} e^2 e_i \cos (2t + 2x - z) + \left\{ \frac{1}{4} + \frac{5}{2} - 5 \right\} e e_i^2 \cos (x + 2z) \\
& \hspace{10cm} [28] \hspace{10cm} [29]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ -\frac{3}{2} - \frac{15}{4} - \frac{15}{2} \right\} e e_i^2 \cos (2t - x - 2z) \\
& \hspace{10cm} [30]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ \frac{5}{4} - \frac{15}{4} + \frac{5}{2} \right\} e e_i^2 \cos (2t + x + 2z) + \left\{ \frac{1}{4} + \frac{5}{2} - 5 \right\} e e_i^2 \cos (x - 2z) \\
& \hspace{10cm} [31] \hspace{10cm} [32]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ -\frac{15}{4} + \frac{45}{4} - \frac{15}{2} \right\} e e_i^2 \cos (2t - x + 2z) + \left\{ \frac{1}{2} + \frac{5}{4} + \frac{5}{2} \right\} e e_i^2 \cos (2t + x - 2z) \\
& \hspace{10cm} [33] \hspace{10cm} [34]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ -\frac{1}{8} - \frac{5}{8} - 5 + \frac{145}{16} \right\} e_i^3 \cos 3z + \left\{ \frac{25}{48} + \frac{5}{4} + \frac{5}{2} + \frac{145}{32} \right\} e_i^3 \cos (2t - 3z) \\
& \hspace{10cm} [35] \hspace{10cm} [36]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ -\frac{7}{48} + \frac{25}{8} - \frac{15}{2} + \frac{145}{32} \right\} e_i^3 \cos (2t + 3z) \\
& \hspace{10cm} [37]
\end{aligned}$$

$$\begin{aligned}
& - \frac{e^4}{12} \cos 4x - \frac{e^4}{32} \cos (2t - 4x) + \frac{9}{16} e^4 \cos (2t + 4x) \\
& \hspace{10cm} [38] \hspace{10cm} [39] \hspace{10cm} [40]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ \frac{1}{8} - \frac{5}{16} \right\} e^3 e_i \cos (3x + z) + \left\{ -\frac{7}{48} - \frac{35}{96} \right\} e^3 e_i \cos (2t - 3x - z) \\
& \hspace{10cm} [41] \hspace{10cm} [42]
\end{aligned}$$

$$\begin{aligned}
& + \left\{ -\frac{25}{16} + \frac{125}{96} \right\} e^3 e_i \cos (2t - 3x - z) + \left\{ \frac{1}{8} - \frac{5}{16} \right\} e^3 e_i \cos (3x - z) \\
& \hspace{10cm} [43] \hspace{10cm} [44]
\end{aligned}$$

$$+ \left\{ \frac{7}{16} - \frac{35}{96} \right\} e^3 e_l \cos (2t - 3x + z)$$

[45]

$$+ \left\{ \frac{25}{48} + \frac{125}{96} \right\} e^3 e_l \cos (2t + 3x - z) + \left\{ \frac{1}{16} + \frac{5}{8} - \frac{5}{4} \right\} e^2 e_l^2 \cos (2x + 2z)$$

[46] [47]

$$+ \left\{ \frac{5}{4} + \frac{25}{8} + \frac{25}{4} \right\} e^2 e_l^2 \cos (2t - 2x - 2z) + \left\{ \frac{5}{4} - \frac{15}{4} + \frac{5}{2} \right\} e^2 e_l^2 \cos (2t + 2x + 2z)$$

[48] [49]

$$+ \left\{ \frac{1}{16} + \frac{5}{8} - \frac{5}{4} \right\} e^2 e_l^2 \cos (2x - 2z) + \left\{ \frac{25}{8} - \frac{75}{8} + \frac{25}{4} \right\} e^2 e_l^2 \cos (2t - 2x + 2z)$$

[50] [51]

$$+ \left\{ \frac{1}{2} + \frac{5}{4} + \frac{5}{2} \right\} e^2 e_l^2 \cos (2t + 2x - 2z) + \left\{ \frac{1}{8} + \frac{5}{8} + 5 - \frac{145}{16} \right\} e e_l^3 \cos (x + 3z)$$

[52] [53]

$$+ \left\{ -\frac{25}{16} - \frac{15}{4} - \frac{15}{2} - \frac{435}{32} \right\} e e_l^3 \cos (2t - x - 3z)$$

[54]

$$+ \left\{ -\frac{7}{48} + \frac{25}{8} - \frac{15}{2} + \frac{145}{32} \right\} e e_l^3 \cos (2t + x + 3z)$$

[55]

$$+ \left\{ \frac{1}{8} + \frac{5}{8} + 5 - \frac{145}{16} \right\} e e_l^3 \cos (x - 3z) + \left\{ \frac{7}{16} - \frac{75}{8} + \frac{45}{2} - \frac{435}{32} \right\} e e_l^3 \cos (2t - x + 3z)$$

[56] [57]

$$+ \left\{ \frac{25}{48} + \frac{5}{4} + \frac{5}{2} - \frac{145}{32} \right\} e e_l^3 \cos (2t + x - 3z) - \left\{ -\frac{1}{12} - \frac{5}{16} - \frac{5}{4} - \frac{145}{16} + \frac{745}{96} \right\} e_l^4 \cos 4z$$

[58] [59]

$$+ \left\{ \frac{9}{16} + \frac{125}{96} + \frac{5}{2} + \frac{145}{32} + \frac{745}{192} \right\} e_l^4 \cos (2t - 4z)$$

[60]

$$+ \left\{ -\frac{1}{32} - \frac{35}{96} + \frac{25}{4} - \frac{435}{32} + \frac{745}{192} \right\} e_l^4 \cos (2t + 4z)$$

[61]

Terms in R multiplied by $-\frac{3}{2} \sin^2 \frac{t}{2} \cos^2 \frac{t}{2} \frac{a^2}{a_l^3}$

$$= \left\{ 1 - \frac{5}{2} e^2 + \frac{3}{2} e_l^2 + 5 e_l^2 - \frac{5}{2} e_l^2 - \frac{5}{2} e_l^2 \right\} \cos 2y$$

[62]

$$+ \left\{ 1 + \frac{3}{2} e^2 - \frac{5}{2} e_l^2 + 5 e_l^2 + \frac{5}{2} e_l^2 - \frac{15}{2} e_l^2 \right\} \cos (2t - 2y)$$

[63]

$$- 3e \cos (x - 2y) + e \cos (x + 2y) - e \cos (2t - x - 2y) - e \cos (2t + x - 2y)$$

[65] [66] [67] [69]

$$+ \left\{ -1 + \frac{5}{2} \right\} e_i \cos (z - 2y) + \left\{ -1 + \frac{5}{2} \right\} e_i \cos (z + 2y) + \left\{ 1 + \frac{5}{2} \right\} e_i \cos (2t - z - 2y) \quad [71] \quad [72] \quad [73]$$

$$+ \left\{ -3 + \frac{5}{2} \right\} e_i \cos (2t + z - 2y) + \frac{5}{2} e^2 \cos (2x - 2y) + e^2 \cos (2x + 2y) \quad [75] \quad [77] \quad [78]$$

$$- \frac{e^2}{4} \cos (2t - 2x - 2y) - \frac{e^2}{4} \cos (2t + 2x - 2y) \quad [79] \quad [81]$$

$$+ \left\{ 3 - \frac{15}{2} \right\} e e_i \cos (x + z - 2y) + \left\{ -1 + \frac{5}{2} \right\} e e_i \cos (x + z + 2y) \quad [83] \quad [84]$$

$$+ \left\{ -1 - \frac{5}{2} \right\} e e_i \cos (2t - x - z - 2y) \quad [85]$$

$$+ \left\{ 3 - \frac{5}{2} \right\} e e_i \cos (2t + x + z - 2y) + \left\{ 3 - \frac{15}{2} \right\} e e_i \cos (2t - z - 2y) \quad [87] \quad [89]$$

$$+ \left\{ -1 + \frac{5}{2} \right\} e e_i \cos (x - z + 2y) + \left\{ 3 - \frac{5}{2} \right\} e e_i \cos (2t - x + z - 2y) \quad [90] \quad [91]$$

$$+ \left\{ -1 - \frac{5}{2} \right\} e e_i \cos (2t + x - z - 2y) + \left\{ -\frac{3}{8} - \frac{5}{2} + 5 \right\} e_i^2 \cos (2z - 2y) \quad [93] \quad [95]$$

$$+ \left\{ -\frac{5}{8} - \frac{5}{2} + 5 \right\} e_i^2 \cos (2z + 2y) + \left\{ \frac{5}{8} + \frac{5}{2} + 5 \right\} e_i^2 \cos (2t - 2z - 2y) \quad [96] \quad [97]$$

$$+ \left\{ \frac{19}{8} - \frac{15}{2} + 5 \right\} e_i^2 \cos (2t + 2z - 2y) \quad [99]$$

Terms in R multiplied by $-\frac{3}{2} \cos^4 \frac{t}{2} \frac{a^2}{a_i^3}$

$$= \frac{1}{2} + \frac{3}{4} e^2 + \frac{3}{4} e_i^2 + \frac{15}{16} e_i^4 + \frac{9}{8} e^2 e_i^2 + \left\{ \frac{1}{2} - \frac{5}{4} e^2 - \frac{5}{4} e_i^2 + \frac{23}{32} e^4 + \frac{13}{32} e_i^4 + \frac{25}{8} e^2 e_i^2 \right\} \cos 2t \quad [1]$$

$$+ \left\{ 1 + \frac{e^2}{8} - \frac{3}{2} e_i^2 \right\} e \cos x + \left\{ -\frac{3}{2} + \frac{13}{16} e^2 + \frac{15}{4} e_i^2 \right\} e \cos (2t - x) \quad [2] \quad [3]$$

$$+ \left\{ \frac{1}{2} - \frac{19}{16} e^2 - \frac{5}{4} e_i^2 \right\} e \cos (2t + x) + \left\{ \frac{3}{2} + \frac{9}{4} e^2 + \frac{27}{16} e_i^2 \right\} e_i \cos z \quad [4] \quad [5]$$

$$+ \left\{ \frac{7}{4} - \frac{35}{8} e^2 - \frac{123}{32} e_l^2 \right\} e_l \cos (2t - z)$$

[6]

$$+ \left\{ -\frac{1}{4} + \frac{5}{8} e^2 + e_l^2 \right\} e_l \cos (2t + z) + \left\{ -\frac{1}{4} + \frac{e^2}{12} - \frac{3}{8} e_l^2 \right\} e^2 \cos 2x$$

[7] [8]

$$+ \left\{ \frac{5}{4} - \frac{25}{8} e_l^2 \right\} e^2 \cos (2t - 2x) + \left\{ \frac{1}{2} - \frac{5}{4} e^2 - \frac{5}{4} e_l^2 \right\} e^2 \cos (2t + 2x)$$

[9] [10]

$$+ \left\{ -\frac{3}{2} + \frac{3}{16} e^2 - \frac{27}{16} e_l^2 \right\} e e_l \cos (x + z) + \left\{ -\frac{21}{4} + \frac{91}{32} e^2 + \frac{369}{32} e_l^2 \right\} e e_l \cos (2t - x - z)$$

[11] [12]

$$+ \left\{ -\frac{1}{4} + \frac{19}{32} e^2 + \frac{e_l^2}{32} \right\} e e_l \cos (2t + x + z) + \left\{ -\frac{3}{2} + \frac{3}{16} e^2 - \frac{27}{16} e_l^2 \right\} e e_l \cos (x - z)$$

[13] [14]

$$+ \left\{ \frac{3}{4} - \frac{13}{32} e^2 - \frac{3}{32} e_l^2 \right\} e e_l \cos (2t - x + z) + \left\{ \frac{7}{4} - \frac{133}{32} e^2 - \frac{123}{32} e_l^2 \right\} e e_l \cos (2t + x - z)$$

[15] [16]

$$+ \left\{ \frac{9}{4} + \frac{27}{8} e^2 + \frac{21}{12} e_l^2 \right\} e_l^2 \cos 2z + \left\{ \frac{17}{4} - \frac{85}{8} e^2 - \frac{115}{12} e_l^2 \right\} e_l^2 \cos (2t - 2z)^* - \frac{e^3}{8} \cos 3x$$

[17] [18] [20]

$$- \frac{7}{48} e^3 \cos (2t - 3x) + \frac{25}{48} e^3 \cos (2t + 3x) - \frac{3}{8} e^2 e_l \cos (2x + z)$$

[21] [22] [23]

$$+ \frac{35}{8} e^2 e_l \cos (2t - 2x - z) - \frac{e^2 e_l}{4} \cos (2t + 2x + z) - \frac{3}{8} e^2 e_l \cos (2x - z)$$

[24] [25] [26]

$$- \frac{5}{8} e^2 e_l \cos (2t - 2x + z) + \frac{7}{4} e^2 e_l \cos (2t + 2x - z)$$

[27] [28]

$$- \frac{9}{4} e e_l^2 \cos (x + 2z) - \frac{51}{4} e e_l^2 \cos (2t - x - 2z)$$

[29] [30]

$$- \frac{9}{4} e e_l^2 \cos (x - 2z) + \frac{17}{4} e e_l^2 \cos (2t + x - 2z)$$

[32] [34]

$$+ \frac{53}{16} e_l^3 \cos 3z + \frac{845}{96} e_l^3 \cos (2t - 3z) + \frac{e^3 e_l}{96} \cos (2t + 3z) - \frac{e^4}{12} \cos 4x - \frac{e^4}{32} \cos (2t - 4x)$$

[35] [36] [37] [38] [39]

* It is remarkable that the coefficient of argument 19 equals zero.

$$+ \frac{9}{16} e^4 \cos(2t + 4x) - \frac{3}{16} e^3 e_i \cos(3x + z)$$

[40]
[41]

$$- \frac{49}{96} e^3 e_i \cos(2t + 3x - z) - \frac{25}{96} e^3 e_i \cos(2t - 3x - z) - \frac{3}{16} e^3 e_i \cos(3x - z)$$

[42]
[43]
[44]

$$+ \frac{7}{96} e^3 e_i \cos(2t - 3x + z) + \frac{175}{96} e^3 e_i \cos(2t + 3x - z) - \frac{9}{16} e^2 e_i^2 \cos(2x + 2z)$$

[45]
[46]
[47]

$$+ \frac{85}{8} e^2 e_i^2 \cos(2t - 2x - 2z) - \frac{9}{16} e^2 e_i^2 \cos(2x - 2z) + \frac{17}{4} e^2 e_i^2 \cos(2t + 2x - 2z)$$

[48]
[50]
[52]

$$- \frac{53}{16} e e_i^3 \cos(x + 3z) - \frac{845}{32} e e_i^3 \cos(2t - x - 3z) + \frac{e e_i^3}{96} \cos(2t + x - 3z)$$

[53]
[54]
[55]

$$- \frac{53}{16} e e_i^3 \cos(x - 3z) - \frac{e e_i^3}{32} \cos(2t - x + 3z) - \frac{25}{96} e e_i^3 \cos(2t + x - 3z)$$

[56]
[57]
[58]

$$- \frac{283}{96} e_i^4 \cos 4z + \frac{2453}{192} e_i^4 \cos(2t - 4z) - \frac{741}{192} e_i^4 \cos(2t + 4z)$$

[59]
[60]
[61]

Terms in R multiplied by $-\frac{3}{2} \sin^2 \frac{t}{2} \cos^2 \frac{t}{2} \frac{a^2}{a_i^3}$ or $-\frac{3}{8} \sin^2 \frac{t}{2} \frac{a^2}{a_i^3}$

$$= \left\{ 1 - \frac{5}{2} e^2 + \frac{3}{2} e_i^2 \right\} \cos 2y + \left\{ 1 + \frac{3}{2} e^2 - \frac{5}{2} e_i^2 \right\} \cos(2t - 2y) - 3e \cos(x - 2y)$$

[62]
[63]
[65]

$$+ e \cos(x + 2y) - e \cos(2t - x - 2y) - e \cos(2t + x - 2y) + \frac{3}{2} e_i \cos(z - 2y) + \frac{3}{2} e_i \cos(z + 2y)$$

[66]
[67]
[69]
[71]
[72]

$$+ \frac{7}{2} e_i \cos(2t - z - 2y) - \frac{e_i}{2} \cos(2t + z - 2y) + \frac{5}{2} e^2 \cos(2x - 2y) + e^2 \cos(2x + 2y)$$

[73]
[75]
[77]
[78]

$$- \frac{e^2}{4} \cos(2t - 2x - 2y) - \frac{e^2}{4} \cos(2t + 2x - 2y) - \frac{9}{2} e e_i \cos(x + z - 2y)$$

[79]
[81]
[83]

$$+ \frac{3}{2} e e_i \cos(x + z + 2y) - \frac{7}{2} e e_i \cos(2t - x - z - 2y) + \frac{e e_i}{2} \cos(2t + x + z - 2y)$$

[84]
[85]
[87]

$$-\frac{9}{2} e e_i \cos (x-z-2 y)+\frac{3}{2} e e_i \cos (x-z+2 y)+\frac{e e_i}{2} \cos (2 t-x+z-2 y)$$

[89]
[90]
[91]

$$-\frac{7}{2} e e_i \cos (2 t+x-z-2 y)+\frac{17}{8} e_i^2 \cos (2 z-2 y)+\frac{15}{8} e_i^2 \cos (2 z+2 y)$$

[93]
[95]
[96]

$$+\frac{65}{8} e_i^2 \cos (2 t-2 z-2 y)-\frac{e_i^2}{8} \cos (2 t+2 z-2 y)$$

[97]
[99]

$$R=m_i\left\{-\frac{1}{r_i}-\frac{1}{4}\left\{1+\frac{3}{2} e^2+\frac{3}{2} e_i^2+\frac{9}{4} e^2 e_i^2+\frac{15}{8} e_i^4-\frac{3}{2} \gamma^2-\frac{9}{4} \gamma^2 e^2-\frac{9}{4} \gamma^2 e_i^2+\frac{39}{8} \gamma^4\right\} \frac{a^2}{a_i^3}\right.$$

[0]

$$-\frac{3}{4}\left\{1-\frac{5}{2} e^2-\frac{5}{2} e_i^2+\frac{23}{16} e^4+\frac{25}{4} e^2 e_i^2+\frac{13}{16} e_i^4\right\} \cos ^4 \frac{t}{2} \frac{a^2}{a_i^3} \cos 2 t$$

[1]

$$+\frac{1}{2}\left\{1-\frac{e^2}{8}-\frac{3}{2} e_i^2-\frac{3}{2} \gamma^2\right\} \frac{a^2}{a_i^3} e \cos x$$

[2]

$$+\frac{9}{4}\left\{1-\frac{13}{24} e^2-\frac{5}{2} e_i^2\right\} \cos ^4 \frac{t}{2} \frac{a^2}{a_i^3} e \cos (2 t-x)$$

[3]

$$-\frac{3}{4}\left\{1-\frac{19}{8} e^2-\frac{5}{2} e_i^2\right\} \cos ^4 \frac{t}{2} \frac{a^2}{a_i^3} e \cos (2 t+x)$$

[4]

$$-\frac{3}{4}\left\{1+\frac{3}{2} e^2+\frac{9}{8} e_i^2-\frac{3}{2} \gamma^2\right\} \frac{a^2}{a_i^3} e_i \cos z$$

[5]

$$-\frac{21}{8}\left\{1-\frac{5}{2} e^2-\frac{123}{56} e_i^2\right\} \cos ^4 \frac{t}{2} \frac{a^2}{a_i^3} e_i \cos (2 t-z)$$

[6]

$$+\frac{3}{8}\left\{1-\frac{5}{2} e^2-4 e_i^2\right\} \cos ^4 \frac{t}{2} \frac{a^2}{a_i^3} e_i \cos (2 t+z)$$

[7]

$$+\frac{1}{8}\left\{1-\frac{e^2}{3}+\frac{3}{2} e_i^2-\frac{3}{2} \gamma^2\right\} \frac{a^2}{a_i^3} e^2 \cos 2 x$$

[8]

$$-\frac{15}{8}\left\{1-\frac{5}{2} e_i^2\right\} \cos ^4 \frac{t}{2} \frac{a^2}{a_i^3} e^2 \cos (2 t-2 x)$$

[9]

$$-\frac{3}{4}\left\{1-\frac{5}{2} e^2-\frac{5}{2} e_i^2\right\} \cos ^4 \frac{t}{2} \frac{a^2}{a_i^3} e^2 \cos (2 t+2 x)$$

[10]

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$$+ \frac{3}{4} \left\{ 1 - \frac{e^2}{8} + \frac{9}{8} e_i^2 - \frac{3}{2} \gamma^2 \right\} \frac{a^2}{a_i^3} e e_i \cos(x+z) \quad [11]$$

$$+ \frac{63}{8} \left\{ 1 - \frac{91}{168} e^2 - \frac{123}{56} e_i^2 \right\} \cos^4 \frac{t}{2} \frac{a^2}{a_i^3} e e_i \cos(2t-x-z) \quad [12]$$

$$+ \frac{3}{8} \left\{ 1 - \frac{19}{8} e^2 - \frac{e_i^2}{8} \right\} \cos^4 \frac{t}{2} \frac{a^2}{a_i^3} e e_i \cos(2t+x+z) \quad [13]$$

$$+ \frac{3}{4} \left\{ 1 - \frac{e^2}{8} + \frac{9}{8} e_i^2 - \frac{3}{2} \gamma^2 \right\} \frac{a^2}{a_i^3} e e_i \cos(x-z) \quad [14]$$

$$- \frac{9}{8} \left\{ 1 - \frac{13}{24} e^2 - \frac{e_i^2}{8} \right\} \cos^4 \frac{t}{2} \frac{a^2}{a_i^3} e e_i \cos(2t-x+z) \quad [15]$$

$$- \frac{21}{8} \left\{ 1 - \frac{19}{8} e^2 - \frac{123}{56} e_i^2 \right\} \cos^4 \frac{t}{2} \frac{a^2}{a_i^3} e e_i \cos(2t+x-z) \quad [16]$$

$$- \frac{9}{8} \left\{ 1 + \frac{3}{2} e^2 + \frac{7}{9} e_i^2 - \frac{3}{2} \gamma^2 \right\} \frac{a^2}{a_i^3} e_i^2 \cos 2z \quad [17]$$

$$- \frac{51}{8} \left\{ 1 - \frac{5}{2} e^2 - \frac{115}{51} e_i^2 \right\} \cos^4 \frac{t}{2} \frac{a^2}{a_i^3} e_i^2 \cos(2t-2z) \quad [18]$$

$$+ \frac{1}{16} \frac{a^2}{a_i^3} e^3 \cos 3x + \frac{7}{32} \frac{a^2}{a_i^3} e^3 \cos(2t-3x) - \frac{25}{32} \frac{a^2}{a_i^3} e^3 \cos(2t+3x) \quad [20] \quad [21] \quad [22]$$

$$+ \frac{3}{16} \frac{a^2}{a_i^3} e^2 e_i \cos(2x+z) - \frac{105}{16} \frac{a^2}{a_i^3} e^2 e_i \cos(2t-2x-z) + \frac{3}{8} \frac{a^2}{a_i^3} e^2 e_i \cos(2t+2x+z) \quad [23] \quad [24] \quad [25]$$

$$+ \frac{3}{16} \frac{a^2}{a_i^3} e^2 e_i \cos(2x-z) + \frac{15}{16} \frac{a^2}{a_i^3} e^2 e_i \cos(2t-2x+z) - \frac{21}{8} \frac{a^2}{a_i^3} e^2 e_i \cos(2t+2x-z) \quad [26] \quad [27] \quad [28]$$

$$+ \frac{9}{8} \frac{a^2}{a_i^3} e e_i^2 \cos(x+2z) + \frac{153}{8} \frac{a^2}{a_i^3} e e_i^2 \cos(2t-x-2z) \quad [29] \quad [30]$$

$$+ \frac{9}{8} \frac{a^2}{a_i^3} e e_i^2 \cos(x-2z) - \frac{51}{8} \frac{a^2}{a_i^3} e e_i^2 \cos(2t+x-2z) - \frac{53}{32} \frac{a^2}{a_i^3} e_i^3 \cos 3x \quad [32] \quad [34] \quad [35]$$

$$- \frac{845}{64} \frac{a^2}{a_i^3} e_i^3 \cos(2t-3x) - \frac{1}{64} \frac{a^2}{a_i^3} e_i^3 \cos(2t+3x) + \frac{1}{24} \frac{a^2}{a_i^3} e^4 \cos 4x \quad [36] \quad [37] \quad [38]$$

$$+ \frac{3}{64} \frac{a^2}{a_i^3} e^4 \cos(2t-4x) - \frac{27}{32} \frac{a^2}{a_i^3} e^4 \cos(2t+4x) + \frac{3}{32} \frac{a^2}{a_i^3} e^3 e_i \cos(3x+z) \quad [39] \quad [40] \quad [41]$$

$$+ \frac{49}{64} \frac{a^2}{a_i^3} e^3 e_i \cos(2t - 3x - z) + \frac{25}{64} \frac{a^2}{a_i^3} e^3 e_i \cos(2t - 3x - z) + \frac{3}{32} \frac{a^2}{a_i^3} e^3 e_i \cos(3x - z) \quad \begin{array}{l} \text{Development} \\ \text{of } R. \end{array}$$

[42]
[44]

$$- \frac{7}{64} \frac{a^2}{a_i^3} e^3 e_i \cos(2t - 3x + z) - \frac{175}{64} \frac{a^2}{a_i^3} e^3 e_i \cos(2t + 3x - z)$$

[45]
[46]

$$+ \frac{9}{32} \frac{a^2}{a_i^3} e^2 e_i^2 \cos(2x + 2z) - \frac{255}{16} e^2 e_i^2 \cos(2t - 2x - 2z)$$

[47]
[48]

$$+ \frac{9}{32} \frac{a^2}{a_i^3} e^2 e_i^2 \cos(2x - 2z) - \frac{51}{8} \frac{a^2}{a_i^3} e^2 e_i^2 \cos(2t + 2x - 2z) + \frac{53}{32} \frac{a^2}{a_i^3} e e_i^3 \cos(x + 3z)$$

[50]
[52]
[53]

$$+ \frac{2535}{64} \frac{a^2}{a_i^3} e e_i^3 \cos(2t - x - 3z) - \frac{1}{64} \frac{a^2}{a_i^3} e e_i^3 \cos(2t + x + 3z)$$

[54]
[55]

$$+ \frac{53}{32} \frac{a^2}{a_i^3} e e_i^3 \cos(x - 3z) + \frac{3}{64} \frac{a^2}{a_i^3} e e_i^3 \cos(2t - x + 3z)$$

[56]
[57]

$$+ \frac{45}{64} \frac{a^2}{a_i^3} e e_i^3 \cos(2t + x - 3z) + \frac{591}{64} \frac{a^2}{a_i^3} e_i^4 \cos 4z$$

[58]
[59]

$$- \frac{2453}{128} \frac{a^2}{a_i^3} e_i^4 \cos(2t - 4z) + \frac{741}{128} \frac{a^2}{a_i^3} e_i^4 \cos(2t + 4z)$$

[60]
[61]

$$- \frac{3}{8} \left\{ 1 - \frac{5}{2} e^2 + \frac{3}{2} e_i^2 \right\} \frac{a^2}{a_i^3} \gamma^2 \cos 2y - \frac{3}{8} \left\{ 1 + \frac{3}{2} e^2 - \frac{5}{2} e_i^2 + \frac{\gamma^2}{8} \right\} \frac{a^2}{a_i^3} \gamma^2 \cos(2t - 2y)$$

[62]
[63]

$$+ \frac{9}{8} \frac{a^2}{a_i^3} \gamma^2 e \cos(x - 2y)$$

[65]

$$- \frac{3}{8} \frac{a^2}{a_i^3} \gamma^2 e \cos(x + 2y) + \frac{3}{8} \frac{a^2}{a_i^3} \gamma^2 e \cos(2t - x - 2y)$$

[66]
[67]

$$+ \frac{3}{8} \frac{a^2}{a_i^3} \gamma^2 e \cos(2t + x - 2y) - \frac{9}{16} \frac{a^2}{a_i^3} \gamma^2 e_i \cos(z - 2y)$$

[69]
[71]

$$- \frac{9}{16} \frac{a^2}{a_i^3} \gamma^2 e_i \cos(z + 2y) - \frac{21}{16} \frac{a^2}{a_i^3} \gamma^2 e_i \cos(2t - z - 2y)$$

[72]
[73]

$$+ \frac{3}{16} \frac{a^2}{a_i^3} \gamma^2 e_i \cos(2t + z - 2y) - \frac{15}{16} \frac{a^2}{a_i^3} \gamma^2 e^2 \cos(2x - 2y)$$

[75]
[77]

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$$-\frac{3}{8} \frac{a^2}{a_i^3} \gamma^2 e^2 \cos(2x + 2y) + \frac{3}{32} \frac{a^2}{a_i^3} \gamma^2 e^2 \cos(2t - 2x - 2y) \quad [78] \quad [79]$$

$$+ \frac{3}{32} \frac{a^2}{a_i^3} \gamma^2 e^2 \cos(2t + 2x - 2y) + \frac{27}{16} \frac{a^2}{a_i^3} \gamma^2 e e_i \cos(x + z - 2y) \quad [81] \quad [83]$$

$$-\frac{9}{16} \frac{a^2}{a_i^3} \gamma^2 e e_i \cos(x + z + 2y) + \frac{21}{16} \frac{a^2}{a_i^3} \gamma^2 e e_i \cos(2t - x - z - 2y) \quad [84] \quad [85]$$

$$-\frac{3}{16} \frac{a^2}{a_i^3} \gamma^2 e e_i \cos(2t + x + z - 2y) + \frac{27}{16} \frac{a^2}{a_i^3} \gamma^2 e e_i \cos(x - z - 2y) \quad [87] \quad [89]$$

$$-\frac{9}{16} \frac{a^2}{a_i^3} \gamma^2 e e_i \cos(x - z + 2y) - \frac{3}{16} \frac{a^2}{a_i^3} \gamma^2 e e_i \cos(2t - x + z - 2y) \quad [90] \quad [91]$$

$$+ \frac{21}{16} \frac{a^2}{a_i^3} \gamma^2 e e_i \cos(2t + x - z - 2y) \quad [93]$$

$$-\frac{51}{64} \frac{a^2}{a_i^3} \gamma^2 e_i^2 \cos(2z - 2y) - \frac{45}{64} \frac{a^2}{a_i^3} \gamma^2 e_i^2 \cos(2z + 2y) \quad [95] \quad [96]$$

$$-\frac{195}{64} \frac{a^2}{a_i^3} \gamma^2 e_i^2 \cos(2t - 2z - 2y) + \frac{3}{64} \frac{a^2}{a_i^3} \gamma^2 e_i^2 \cos(2t + 2z - 2y) \quad [97] \quad [99]$$

$$-\frac{3}{8} \left\{ 1 + 3e^2 + 3e_i^2 - \frac{11}{4} \gamma^2 \right\} \frac{a^3}{a_i^4} \cos t + \frac{15}{16} \frac{a^3}{a_i^4} e \cos(t - x) \quad [101] \quad [102]$$

$$+ \frac{3}{16} \frac{a^3}{a_i^4} e \cos(t + x) - \frac{9}{8} \frac{a^3}{a_i^4} e_i \cos(t - z) - \frac{3}{8} \frac{a^3}{a_i^4} e_i \cos(t + x) \quad [103] \quad [104] \quad [105]$$

$$-\frac{33}{64} \frac{a^3}{a_i^4} e^2 \cos(t - 2x) + \frac{9}{64} \frac{a^3}{a_i^4} e^2 \cos(t + 2x) + \frac{45}{16} \frac{a^3}{a_i^4} e e_i \cos(t - x - z) \quad [106] \quad [107] \quad [108]$$

$$+ \frac{3}{16} \frac{a^3}{a_i^4} e e_i \cos(t + x + z) + \frac{15}{16} \frac{a^3}{a_i^4} e e_i \cos(t - x + z) + \frac{9}{16} \frac{a^3}{a_i^4} e e_i \cos(t + x - z) \quad [109] \quad [110] \quad [111]$$

$$-\frac{159}{64} \frac{a^3}{a_i^4} e_i^2 \cos(t - 2z) - \frac{33}{64} \frac{a^3}{a_i^4} e_i^2 \cos(t + 2z) - \frac{9}{16} \frac{a^3}{a_i^4} \gamma^2 \cos(t - 2y) \quad [112] \quad [113] \quad [114]$$

$$-\frac{15}{32} \frac{a^3}{a_i^4} \sin^2 \frac{t}{2} \cos(t + 2y) - \frac{5}{8} \left\{ 1 - 6e^2 - 6e_i^2 - \frac{3}{4} \gamma^2 \right\} \frac{a^3}{a_i^4} \cos 3t \quad [115] \quad [116]$$

* For the coefficients of the terms multiplied by $\frac{a^3}{a_i^4}$ see p. 39.

$$\begin{aligned}
& + \frac{45}{16} \frac{a^3}{a_i^4} e \cos(3t - x) - \frac{15}{16} \frac{a^3}{a_i^4} e \cos(3t + x) - \frac{25}{8} \frac{a^3}{a_i^4} e_i \cos(3t - z) \\
& \quad [117] \qquad [118] \qquad [119] \\
& + \frac{5}{8} \frac{a^3}{a_i^4} e_i \cos(3t + z) - \frac{285}{64} \frac{a^3}{a_i^4} e^2 \cos(3t - 2x) - \frac{75}{64} \frac{a^3}{a_i^4} e^2 \cos(3t + 2x) \\
& \quad [120] \qquad [121] \qquad [122] \\
& - \frac{225}{16} \frac{a^3}{a_i^4} e e_i \cos(3t - x - z) + \frac{15}{16} \frac{a^3}{a_i^4} e e_i \cos(3t + x + z) \\
& \quad [123] \qquad [124] \\
& - \frac{45}{16} \frac{a^3}{a_i^4} e e_i \cos(3t - x + z) - \frac{75}{16} \frac{a^3}{a_i^4} e e_i \cos(3t + x - z) \\
& \quad [125] \qquad [126] \\
& - \frac{635}{64} \frac{a^3}{a_i^4} e_i^2 \cos(3t - 2z) - \frac{5}{64} \frac{a^3}{a_i^4} e_i^2 \cos(3t + 2z) - \frac{15}{32} \frac{a^3}{a_i^4} \gamma^2 \cos(3t - 2y) \\
& \quad [127] \qquad [128] \qquad [129]
\end{aligned}$$

In the elliptic movement ;

$$s = \gamma \sin(g\lambda - \nu)$$

$$\lambda = nt + 2e \sin x + \frac{5}{4} e^2 \sin 2x$$

$$\begin{aligned}
s &= \gamma(1 - e^2) \sin y + \gamma e \sin(x - y) + \gamma e \sin(x + y) + \gamma \frac{e^2}{8} \sin(2x - y) + \frac{9}{8} \gamma e^2 \sin(2x + y) \\
& \quad [146] \qquad [149] \qquad [150] \qquad [161] \qquad [162]
\end{aligned}$$

$$\begin{aligned}
s^2 &= \frac{\gamma^2}{2} - \frac{\gamma^2}{2} (1 - 4e^2) \cos 2y + \gamma^2 e \cos(x - 2y) - \gamma^2 e \cos(x + 2y) \\
& \quad [62] \qquad [65] \qquad [66]
\end{aligned}$$

$$\begin{aligned}
& + \frac{5}{8} \gamma^2 e^2 \cos(2x - 2y) - \frac{5}{8} \gamma^2 e^2 \cos(2x + 2y) \\
& \quad [77] \qquad [78]
\end{aligned}$$

$$\begin{aligned}
z^* &= a\gamma \left(1 - \frac{e^2}{2}\right) \sin y + \frac{3a\gamma e}{2} \sin(x - y) + \frac{a\gamma e}{2} \sin(x + y) \\
& \quad [146] \qquad [149] \qquad [150]
\end{aligned}$$

$$\begin{aligned}
& - \frac{a\gamma e^2}{8} \sin(2x - y) + \frac{3a\gamma e^2}{8} \sin(2x + y) \\
& \quad [161] \qquad [162]
\end{aligned}$$

$$\begin{aligned}
\frac{s}{r} &= \frac{\gamma}{a} (1 - e^2) \sin y + \frac{\gamma e}{2a} \sin(x - y) + \frac{3\gamma e}{2a} \sin(x + y) \\
& \quad [146] \qquad [149] \qquad [150]
\end{aligned}$$

* This quantity z , which is one of the rectangular coordinates of the moon, must not be confounded with $z = n_i t - \varpi_i$; this latter quantity should rather be x_i , but I think it better to conform as far as possible to the notation of M. DAMOISEAU.

$$+ \frac{\gamma e^2}{8a} \sin(2x-y) + \frac{17}{8} \frac{\gamma e^2}{a} \sin(2x+y)$$

[161]
[162]

$$\frac{s}{r} \delta \cdot \frac{1}{r} = \left\{ (1-e^2)r_0 + \frac{e^2}{2}r_2 \right\} \frac{\gamma}{a^2} \sin y - \left\{ (1-e^2)\frac{r_1}{2} + \frac{e^2}{4}r_3 - \frac{3e^2}{4}r_4 \right\} \frac{\gamma}{a^2} \sin(2t-y)$$

[146]
[147]

$$+ \left\{ (1-e^2)\frac{r_1}{2} - \frac{e^2}{4}r_4 + \frac{3e^2}{4}r_3 \right\} \frac{e\gamma}{a^2} \sin(2t+y) + \frac{e\gamma r_0}{2a^2} \sin(x-y)$$

[148]
[149]

$$+ \frac{3r_0}{2a^2} e\gamma \sin(x+y) + \left\{ -\frac{r_3}{2} - \frac{3r_1}{4} \right\} \frac{e\gamma}{a^2} \sin(2t-x-y)$$

[150]
[151]

$$+ \left\{ \frac{r_3}{2} - \frac{r_1}{4} \right\} \frac{e\gamma}{a^2} \sin(2t-x+y) + \left\{ -\frac{r_4}{2} - \frac{r_1}{4} \right\} \frac{e\gamma}{a^2} \sin(2t+x-y)$$

[152]
[153]

$$+ \left\{ \frac{r_4}{2} + \frac{3}{4}r_1 \right\} \frac{e\gamma}{a^2} \sin(2t+x+y) + \frac{r_5 e_l \gamma}{2a^2} \sin(z-y) + \frac{r_5 e_l \gamma}{2a^2} \sin(z+y)$$

[154]
[155]
[156]

$$- \frac{r_6 e_l \gamma}{2a^2} \sin(2t-z-y) + \frac{r_6 e_l \gamma}{2a^2} \sin(2t-z+y) - \frac{r_7 e_l \gamma}{2a^2} \sin(2t+z-y)$$

[157]
[158]
[159]

$$+ \left\{ -\frac{r_9}{2} - \frac{3}{4}r_3 - \frac{17}{16}r_1 \right\} \frac{e^2 \gamma}{a^2} \sin(2t-2x-y) + \left\{ \frac{r_9}{2} - \frac{r_3}{4} - \frac{r_1}{16} \right\} \frac{e^2 \gamma}{a^2} \sin(2t-2x+y)$$

[163]
[164]

$$+ \left\{ -\frac{r_{10}}{2} + \frac{r_4}{4} + \frac{r_1}{16} \right\} \frac{e^2 \gamma}{a^2} \sin(2t+2x-y) + \left\{ \frac{r_{10}}{2} + \frac{3r_4}{4} + \frac{17}{16}r_1 \right\} \frac{e^2 \gamma}{a^2} \sin(2t+2x+y)$$

[165]
[166]

$$+ \left\{ -\frac{r_{11}}{2} + \frac{r_5}{4} \right\} \frac{e e_l \gamma}{a^2} \sin(x+z-y) + \left\{ \frac{r_{11}}{2} + \frac{3r_5}{4} \right\} \frac{e e_l \gamma}{a^2} \sin(x+z+y)$$

[167]
[168]

$$+ \left\{ -\frac{r_{12}}{2} + \frac{r_6}{4} - \frac{3}{4}r_6 \right\} \frac{e e_l \gamma}{a^2} \sin(2t-x-z-y) + \left\{ \frac{r_{12}}{2} + \frac{r_6}{4} \right\} \frac{e e_l \gamma}{a^2} \sin(2t-x-z+y)$$

[169]
[170]

$$+ \left\{ -\frac{r_{13}}{2} + \frac{r_7}{4} \right\} \frac{e e_l \gamma}{a^2} \sin(2t+x+z-y) + \left\{ \frac{r_{13}}{2} + \frac{3}{4}r_7 \right\} \frac{e e_l \gamma}{a^2} \sin(2t+x+z+y)$$

(171)
(172)

$$+ \left\{ -\frac{r_{14}}{2} + \frac{r_5}{2} \right\} \frac{e e_l \gamma}{a^2} \sin(x-z-y) + \left\{ \frac{r_{14}}{2} + \frac{3}{4}r_5 \right\} \frac{e e_l \gamma}{a^2} \sin(x-z+y)$$

(173)
(174)

$$+ \left\{ -\frac{r_{15}}{2} - \frac{3}{4}r_7 \right\} \frac{e e_l \gamma}{a^2} \sin(2t-x+z-y) + \left\{ \frac{r_{15}}{2} - \frac{r_7}{2} \right\} \frac{e e_l \gamma}{a^2} \sin(2t-x+z+y)$$

[175]
[176]

$$-\frac{r_{16} e e_i \gamma}{2 a^2} \sin (2 t+x-z-y)+\left\{\frac{r_{16}}{2}+\frac{3}{4} r_6\right\} \frac{e e_i \gamma}{a^2} \sin (2 t+x-z+y) \quad [177] \quad [178]$$

$$-\frac{r_{17} e_i^2 \gamma}{2 a^2} \sin (2 z-y)+\frac{r_{17} e_i^2 \gamma}{2 a^2} \sin (2 z+y)-\frac{r_{18} e_i^2 \gamma}{2 a^2} \sin (2 t-2 z-y) \quad [179] \quad [180] \quad [181]$$

$$+\frac{r_{18} e_i^2 \gamma}{2 a^2} \sin (2 t-2 z+y)-\frac{r_{19} e_i^2 \gamma}{2 a^2} \sin (2 t+2 z-y)+\frac{r_{19} e_i^2 \gamma}{2 a^2} \sin (2 t+2 z+y) \quad [182] \quad [183] \quad [184]$$

$$\frac{m_i z}{r^3}=\frac{m_i a \gamma}{a_i^3}\left(1+\frac{3}{2} e_i^2-\frac{e^2}{2}\right) \sin y+\frac{3 m_i a \gamma e}{2 a_i^3} \sin (x-y)+\frac{m_i a \gamma}{2 a_i^3} \sin (x+y) \quad [146] \quad [149] \quad [150]$$

$$-\frac{3 m_i a \gamma e_i}{2 a_i^3} \sin (z-y)+\frac{3 m_i a \gamma e_i}{2 a_i^3} \sin (z+y)-\frac{m_i a \gamma e^2}{8 a_i^3} \sin (2 x-y) \quad [155] \quad [156] \quad [161]$$

$$+\frac{3 m_i a \gamma e^2}{8 a_i^3} \sin (2 x+y)+\frac{9 m_i a \gamma e e_i}{4 a_i^3} \sin (x+z-y)+\frac{3 m_i a \gamma e e_i}{4 a_i^3} \sin (x+z+y) \quad [162] \quad [167] \quad [168]$$

$$+\frac{9 m_i a \gamma e e_i}{4 a_i^3} \sin (x-z-y)+\frac{3 m_i a \gamma e e_i}{4 a_i^3} \sin (x-z+y)-\frac{9 m_i a \gamma e_i^2}{4 a_i^3} \sin (2 z-y) \quad [173] \quad [174] \quad [179]$$

$$+\frac{9 m_i a \gamma e_i^2}{4 a_i^3} \sin (2 z+y) \quad [180]$$

$$\frac{a^3}{r^3}=1+\frac{3}{2} e^2+3 e \cos x+\frac{9}{2} e^2 \cos 2 x$$

r being the elliptic value of r .

If $z=a \gamma z_{146} \sin y+a \gamma z_{147} \sin (2 t-y)+a \gamma z_{148} \sin (2 t+y) \&c$.

$$\frac{z}{r^3}^*=\left\{\left(1+\frac{3 e^2}{2}\right) z_{146}+\frac{3}{2} e^2 z_{150}-\frac{3 e^2}{2} z_{149}\right\} \frac{\gamma}{a^2} \sin y \quad [146]$$

$$+\left\{\left(1+\frac{3 e^2}{2}\right) z_{147}+\frac{3 e^2}{2} z_{151}+\frac{3 e^2}{2} z_{153}\right\} \frac{\gamma}{a^2} \sin (2 t-y) \quad [147]$$

$$+\left\{\left(1+\frac{3 e^2}{2}\right) z_{148}+\frac{3 e^2}{2} z_{152}+\frac{3 e^2}{2} z_{154}\right\} \frac{\gamma}{a^2} \sin (2 t+y) \quad [148]$$

* This multiplication of z by r^{-3} may be effected at once by means of Table II.

$$+ \left\{ z_{149} - \frac{3}{2} z_{146} \right\} \frac{e\gamma}{a^2} \sin(x-y) + \left\{ z_{150} + \frac{3}{2} z_{146} \right\} \frac{e\gamma}{a^2} \sin(x+y)$$

[149]
[150]

$$+ \left\{ z_{151} + \frac{3}{2} z_{147} \right\} \frac{e\gamma}{a^2} \sin(2t-x-y) + \left\{ z_{152} + \frac{3}{2} z_{148} \right\} \frac{e\gamma}{a^2} \sin(2t-x+y)$$

[151]
[152]

$$+ \left\{ z_{153} + \frac{3}{2} z_{147} \right\} \frac{e\gamma}{a^2} \sin(2t+x-y)$$

[153]

$$+ \left\{ z_{154} + \frac{3}{2} z_{148} \right\} \frac{e\gamma}{a^2} \sin(2t+x+y) + z_{155} \frac{e\gamma}{a^2} \sin(z-y) + z_{156} \frac{e\gamma}{a^2} \sin(z+y)$$

[154]
[155]
[156]

$$+ \left\{ z_{161} + \frac{3}{2} z_{149} - \frac{9}{4} z_{146} \right\} \frac{e^2\gamma}{a^2} \sin(2x-y) + \left\{ z_{162} + \frac{3}{2} z_{150} + \frac{9}{4} z_{146} \right\} \frac{e^2\gamma}{a^2} \sin(2x+y)$$

[161]
[162]

$$+ \left\{ z_{163} + \frac{3}{2} z_{151} + \frac{9}{4} z_{147} \right\} \frac{e^2\gamma}{a^2} \sin(2t-2x-y)$$

[163]

$$+ \left\{ z_{164} + \frac{3}{2} z_{152} + \frac{9}{4} z_{148} \right\} \frac{e^2\gamma}{a^2} \sin(2t-2x+y)$$

[164]

$$+ \left\{ z_{165} + \frac{3}{2} z_{153} + \frac{9}{4} z_{147} \right\} \frac{e^2\gamma}{a^2} \sin(2t+2x-y)$$

[165]

$$+ \left\{ z_{166} + \frac{3}{2} z_{154} + \frac{9}{4} z_{148} \right\} \frac{e^2\gamma}{a^2} \sin(2t+2x+y) + \&c.$$

[166]

$$+ \left\{ z_{167} + \frac{3}{2} z_{155} \right\} \frac{ee_l\gamma}{a^2} \sin(x+z-y) + \left\{ z_{168} + \frac{3}{2} z_{156} \right\} \frac{ee_l\gamma}{a^2} \sin(x+z+y)$$

[167]
[168]

$$+ \left\{ z_{169} + \frac{3}{2} z_{157} \right\} \frac{ee_l\gamma}{a^2} \sin(2t-x-z-y) + \left\{ z_{170} + \frac{3}{2} z_{158} \right\} \frac{ee_l\gamma}{a^2} \sin(2t-x-z+y)$$

[169]
[170]

$$+ \left\{ z_{171} + \frac{3}{2} z_{159} \right\} \frac{ee_l\gamma}{a^2} \sin(2t+x+z-y) + \left\{ z_{172} + \frac{3}{2} z_{160} \right\} \frac{ee_l\gamma}{a^2} \sin(2t+x+z+y)$$

[171]
[172]

$$+ \left\{ z_{173} - \frac{3}{2} z_{156} \right\} \frac{ee_l\gamma}{a^2} \sin(x-z-y) + \left\{ z_{174} - \frac{3}{2} z_{155} \right\} \frac{ee_l\gamma}{a^2} \sin(x-z+y)$$

[173]
[174]

$$+ \left\{ z_{175} + \frac{3}{2} z_{159} \right\} \frac{ee_l\gamma}{a^2} \sin(2t-x+z-y) + \left\{ z_{176} + \frac{3}{2} z_{160} \right\} \frac{ee_l\gamma}{a^2} \sin(2t-x+z+y)$$

[175]
[176]

$$+ \left\{ z_{177} + \frac{3}{2} z_{157} \right\} \frac{ee_l\gamma}{a^2} \sin(2t+x-z-y) + \left\{ z_{178} + \frac{3}{2} z_{158} \right\} \frac{ee_l\gamma}{a^2} \sin(2t+x-z+y)$$

[177]
[178]

$$s = \frac{z}{r} \text{ nearly,}$$

$$\begin{aligned} &= \left\{ z_{146} + \frac{e^2}{2} z_{150} + \frac{e^2}{2} z_{149} \right\} \gamma \sin y \\ &+ \left\{ z_{147} + \frac{e^2}{2} z_{151} + \frac{e^2}{2} z_{153} \right\} \gamma \sin (2t - y) \\ &+ \left\{ z_{148} + \frac{e^2}{2} z_{152} + \frac{e^2}{2} z_{154} \right\} \gamma \sin (2t + y) + \&c. \end{aligned}$$

$$\frac{d^2 \cdot r^2}{2 \cdot d t^2} - \frac{\mu}{r} + \frac{\mu}{a} + 2 \int d R + r \left(\frac{d R}{d r} \right) = 0$$

$$\frac{d^2 z}{d t^2} + \frac{\mu z}{r^3} + \frac{m_i z}{\{r^2 - 2 r r' \cos (\lambda - \lambda) + r_i^2\}^{\frac{3}{2}}}$$

$$r^4 \cdot \frac{d \lambda'^2}{d t^2} = h^2 - 2 \int r^2 \left(\frac{d R}{d \lambda'} \right) d \lambda'$$

Neglecting the square of the disturbing force

$$\frac{-d^2 \cdot r^3 \delta \cdot \frac{1}{r}}{d t^2} - \mu \delta \cdot \frac{1}{r} + 2 \int d R + r \left(\frac{d R}{d r} \right) = 0$$

$$\frac{d^2 z}{d t^2} + \frac{\mu z}{r^3} + \frac{m_i z}{r_i^3} + \frac{3 m_i z r' r \cos (\lambda' - \lambda)}{r_i^5} = 0$$

$$\frac{d^2 \cdot \delta z}{d t^2} + \frac{3 \mu s \delta \cdot \frac{1}{r}}{r} + \frac{\mu \delta \cdot z}{r^3} + \frac{m_i z}{r_i^3} + \frac{3 \mu_i z r' r \cos (\lambda' - \lambda)}{r_i^5} = 0$$

$$\frac{d \lambda'}{d t} = \frac{h(1+s^2)}{r^2} - \frac{(1+s^2)}{r^2} \int \left(\frac{d R}{d \lambda'} \right) d t$$

$$r \left(\frac{d R}{d r} \right) = a \left(\frac{d R}{d a} \right), \quad \frac{d R}{d \lambda'} = \frac{d R}{d t}, \quad (t \text{ being used for } n t - n_i t).$$

Integrating the equation of p. 270, line 9, by the method of indeterminate coefficients, neglecting the cubes and higher powers of e in order to obtain a first approximation, m being equal to $\frac{n_i}{n}$ as in the notation of M. DAMOISEAU ;

$$-r_0 - \frac{m_i a^3}{2 \mu a_i^3} \left\{ 1 + \frac{3}{2} e^2 + \frac{3}{2} e_i^2 - \frac{3}{2} \gamma^2 \right\} = 0$$

$$4(1-m)^2 \left\{ (1+3e^2) r_1 - \frac{3e^2}{2} \{r_3 + r_4\} \right\} - r_1$$

$$-\frac{3m_1a^3}{2\mu a_1^3}\left\{1-\frac{5}{2}e^2-\frac{5}{2}e_1^2-\frac{\gamma^2}{2}\right\}\left\{\frac{1}{1-m}+1\right\}=0$$

$$c^2* \{1-3r_0\}-1+\frac{2m_1a^3}{\mu a^3}=0$$

$$(2-2m-c)^2\left\{r_3-\frac{3}{2}r_1\right\}-r_3+\frac{9}{2}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{2-c}{2-2m-c}+1\right\}=0$$

$$(2-2m+c)^2\left\{r_4-\frac{3}{2}r_1\right\}-r_4-\frac{3}{2}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{2+c}{2-2m+c}+1\right\}=0$$

$$m^2r_5-r_5-\frac{3}{2}\frac{m_1}{\mu}\frac{a^3}{a_1^3}=0$$

$$(2-3m)^2r_6-r_6-\frac{21}{4}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{2}{2-3m}+1\right\}=0$$

$$(2-m)^2r_7-r_7+\frac{3}{4}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{2}{2-m}+1\right\}=0$$

$$4c^2\left\{(1-3r_0)r_8-\frac{3}{4}+3r_0\right\}-r_8+\frac{m_1}{2\mu}\frac{a^3}{a_1^3}=0$$

$$(2-2m-2c)^2\left\{r_9-\frac{3}{2}r_3\right\}-r_9-\frac{15}{4}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{2-2c}{2-2m-2c}+1\right\}=0$$

$$(2-2m+2c)^2\left\{r_{10}-\frac{3}{2}r_4\right\}-r_{10}-\frac{3}{2}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{2+2c}{2-2m+2c}+1\right\}=0$$

$$(c+m)^2\left\{r_{11}-\frac{3}{2}r_5\right\}-r_{11}+\frac{3}{2}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{c}{c+m}+1\right\}=0$$

$$(2-3m-c)^2\left\{r_{12}-\frac{3}{2}r_6\right\}-r_{12}+\frac{63}{4}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{c}{2-3m-c}+1\right\}=0$$

$$(2-m+c)^2\left\{r_{13}-\frac{3}{2}r_7\right\}-r_{13}+\frac{3}{4}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{2+c}{2-m+c}+1\right\}=0$$

$$(c-m)^2\left\{r_{14}-\frac{3}{2}r_5\right\}-r_{14}+\frac{3}{2}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{c}{c-m}+1\right\}=0$$

$$(2-m-c)^2\left\{r_{15}-\frac{3}{2}r_7\right\}-r_{15}-\frac{9}{4}\frac{m_1}{\mu}\frac{a^3}{a_1^3}\left\{\frac{2-c}{2-m-c}+1\right\}=0$$

* The letter c does not strictly denote the same quantity as in the notation of M. DAMOISEAU, or in that of the Mathematical Tracts, p. 33.

$$(2-3m+c)^2 \left\{ r_{16} - \frac{3}{2} r_6 \right\} - r_{16} - \frac{21}{4} \frac{m_l}{\mu} \frac{a^3}{a_l^3} \left\{ \frac{2+c}{2-3m+c} + 1 \right\} = 0$$

$$4m^2 r_{17} - r_{17} - \frac{9}{4} \frac{m_l}{\mu} \frac{a^3}{a_l^3} = 0$$

$$(2-4m)^2 r_{18} - r_{18} - \frac{51}{4} \frac{m_l}{\mu} \frac{a^3}{a_l^3} \left\{ \frac{2}{2-4m} + 1 \right\}$$

$$4r_{19} - r_{19} = 0$$

The equation for determining z may be integrated in the same way.

$$-g^2 z_{146} + 3r_0 + z_{146} + \frac{m_l}{\mu} \frac{a^3}{a_l^3} = 0$$

$$-\left\{ 2(1-m) - g \right\}^2 z_{147} - \frac{3r_1}{2} + z_{147} = 0$$

$$-\left\{ 2(1-m) + g \right\}^2 z_{148} + \frac{3r_1}{2} + z_{148} = 0$$

$$-\left\{ c - g \right\}^2 z_{149} + \frac{3}{2} r_6 + z_{149} - \frac{3}{2} z_{146} + \frac{3m_l}{2\mu} \frac{a^3}{a_l^3} = 0$$

$$-\left\{ c + g \right\}^2 z_{150} + \frac{9}{2} r_6 + z_{150} + \frac{3}{2} z_{146} + \frac{m_l}{\mu} \frac{a^3}{a_l^3} = 0$$

$$-\left\{ 2(1-m) - c - g \right\}^2 z_{147} + 3 \left\{ -\frac{3r_1}{4} - \frac{r_3}{2} \right\} + z_{151} + \frac{3}{2} z_{147} = 0$$

$$-\left\{ 2(1-m) - c + g \right\}^2 z_{148} + 3 \left\{ -\frac{r_1}{4} + \frac{r_3}{2} \right\} + z_{152} + \frac{3}{2} z_{148} = 0$$

$$-\left\{ 2(1-m) + c - g \right\}^2 z_{149} + 3 \left\{ \frac{r_1}{4} - \frac{r_4}{2} \right\} + z_{153} + \frac{3}{2} z_{147} = 0$$

$$-\left\{ 2(1-m) + c + g \right\}^2 z_{150} + 3 \left\{ \frac{3}{4} r_1 + \frac{r_4}{2} \right\} + z_{154} + \frac{3}{2} z_{148} = 0$$

$$-\left\{ m - g \right\}^2 z_{151} + \frac{3}{2} r_5 + z_{155} - \frac{3m_l}{2\mu} \frac{a^3}{a_l^3} = 0$$

$$-\left\{ m + g \right\}^2 z_{152} + \frac{3}{2} r_5 + z_{156} + \frac{3m_l}{2\mu} \frac{a^3}{a_l^3} = 0$$

$$-\left\{ 2(1-m) - m - g \right\}^2 z_{153} - \frac{3}{2} r_6 + z_{157} = 0$$

$$-\left\{ 2(1-m) - m + g \right\}^2 z_{154} + \frac{3}{2} r_6 + z_{158} = 0$$

$$-\left\{ 2(1-m) + m - g \right\}^2 z_{155} - \frac{3}{2} r_7 + z_{159} = 0$$

$$-\left\{2(1-m)+m+g\right\}^2 z_{156}+\frac{3}{2} r_7+z_{160}=0$$

$$\frac{d \lambda'}{d t}=\frac{h}{r^2}+\frac{2 h}{r} \delta . \frac{1}{r}+\frac{h z^2}{r^4}-\frac{(1+s^2)}{r^2} \int\left(\frac{d R}{d \lambda'}\right) d t$$

$$\begin{aligned} \lambda' &= \frac{h}{a^2}\left\{1+\frac{e^2}{2}+\frac{\gamma^2}{2}+2 r_0\right\} t+\frac{2 e\left(1+r_0\right)}{c} \sin x+\frac{5 e^2\left(1+r_0\right)}{4 c} \sin 2 x \\ &+ \left\{2 r_1+e^2\left(r_3+r_4\right)-\left\{-\left(1-\frac{5}{2} e^2-\frac{5}{2} e_l^2-\frac{\gamma^2}{2}\right) \frac{3}{4(1-m)}+\frac{9 e^2}{2(2-2 m-c)}\right.\right. \\ &\quad \left.\left.-\frac{3 e^2}{2(2-2 m+c)}\right\} \frac{m_l a^3}{\mu a_l^3}\right\} \frac{1}{2(1-m)} \sin 2 t \\ &+ \left\{2 r_3+e^2 r_1-\left\{\frac{9}{2(2-m-c)}-\frac{3}{2(2-m)}\right\} \frac{m_l a^3}{\mu a_l^3}\right\} \frac{e}{(2-2 m-c)} \sin (2 t-x) \\ &+ \left\{2 r_4+e^2 r_1-\left\{-\frac{3}{2(2-m+c)}-\frac{3}{2(2-m)}\right\} \frac{m_l a^3}{\mu a_l^3}\right\} \frac{e}{(2-m+c)} \sin (2 t+x) \\ &+ \frac{2 r_5}{m} \sin z \\ &+ \left\{2 r_6+\frac{21}{4(2-3 m)} \frac{m_l a^3}{\mu a_l^3}\right\} \frac{e_l}{(2-3 m)} \sin (2 t-z) \\ &+ \left\{2 r_7-\frac{3}{4(2-m)} \frac{m_l a^3}{\mu a_l^3}\right\} \frac{e_l}{(2-m)} \sin (2 t+z) \\ &+ \left\{2 r_9+r_3-\left\{-\frac{15}{4(2-2 m-2 c)}+\frac{9}{2(2-2 m-c)}\right\} \frac{m_l a^3}{\mu a_l^3}\right\} \frac{e^2}{2(1-m-c)} \sin (2 t-2 x) \\ &+ \left\{2 r_{10}+r_4-\left\{-\frac{3}{2(2-2 m+2 c)}-\frac{3}{2(2-m+c)}\right\} \frac{m_l a^3}{\mu a_l^3}\right\} \frac{e^2}{2(1-m+c)} \sin (2 t+2 x) \\ &+ \left\{2 r_{11}+r_5\right\} \frac{e e_l}{(c+m)} \sin (x+z) \\ &+ \left\{2 r_{12}+r_6-\left\{\frac{63}{4(2-3 m-c)}-\frac{21}{4(2-3 m)}\right\} \frac{m_l a^3}{\mu a_l^3}\right\} \frac{e e_l}{(2-3 m-c)} \sin (2 t-x-z) \\ &+ \left\{2 r_{13}+r_7-\left\{\frac{3}{4(2-m+c)}+\frac{3}{4(2-m)}\right\} \frac{m_l a^3}{\mu a_l^3}\right\} \frac{e e_l}{(2-m+c)} \sin (2 t+x+z) \end{aligned}$$

Considering the terms which depend on the square of the disturbing force

$$\frac{d^2 . r^2}{2 d t^2}-\frac{\mu}{r}+\frac{\mu}{a}+2 \int d R+r\left(\frac{d R}{d r}\right)=0$$

$$\frac{d^2 \cdot r^2}{2 dt^2} - \frac{d^2 \cdot r^3 \delta \cdot \frac{1}{r}}{dt^2} + \frac{3 d^2 \cdot r^4 \left(\delta \cdot \frac{1}{r} \right)^2}{2 dt^2} - \frac{\mu}{r} + \frac{\mu}{a} + 2 \int dR + r \left(\frac{dR}{dr} \right) = 0$$

$$\frac{d^2 z}{dt^2} + \frac{\mu z}{r^3} + \frac{m_i z}{r_i^3} - \frac{3 m_i z r r \cos(\lambda' - \lambda)}{r_i^5} = 0.$$

$$\begin{aligned} \frac{d\lambda'}{dt} &= \frac{h}{r^2} \left\{ 1 - \frac{1}{h} \int \left(\frac{dR}{d\lambda'} \right) dt \left\{ 1 - \frac{1}{h^2} \int \left(\frac{dR}{d\lambda'} \right) dt \right\} - \frac{1}{2h^2} \left\{ \int \left(\frac{dR}{d\lambda'} \right) dt \right\}^2 \right. \\ &= \frac{h(1+s^2)}{r^2} - \frac{(1+s^2)}{r^2} \int \left(\frac{dR}{d\lambda'} \right) dt + \frac{(1+s^2)}{2r^2 h} \left\{ \int \left(\frac{dR}{d\lambda'} \right) dt \right\}^2 \end{aligned}$$

dR = the differential of R , supposing nt only variable + the differential of R , with regard to $n_i t$ only in as much as it is contained in the terms in r, λ and s due to the perturbations; hence

dR = the differential of R , supposing only nt variable + $\frac{dR}{dr} \cdot d \cdot \delta r + \frac{dR}{d\lambda} d \cdot \delta \lambda + \frac{dR}{ds} d \cdot \delta s$, being restrained to mean the differentials of those quantities with regard to $n_i t$ only.

$$\delta R = \left(\frac{dR}{dr} \right) \delta r + \left(\frac{dR}{d\lambda} \right) \delta \lambda + \left(\frac{dR}{ds} \right) \delta s = -a \left(\frac{dR}{da} \right) r \delta \cdot \frac{1}{r} + \left(\frac{dR}{dt} \right) \delta \lambda + \left(\frac{dR}{ds} \right) \delta s,$$

(t being used in the sense $nt - n_i t$.) $\left(\frac{dR}{ds} \right) \delta s = \frac{r^2}{2r_i^3} s \delta s$ nearly.

$$\left(\frac{dR}{dr} \right) d \cdot \delta r + \left(\frac{dR}{d\lambda} \right) d \cdot \delta \lambda + \left(\frac{dR}{ds} \right) d \cdot \delta s = -a \left(\frac{dR}{da} \right) d \cdot r \delta \cdot \frac{1}{r} + \left(\frac{dR}{dt} \right) d \cdot \delta \lambda + \left(\frac{dR}{ds} \right) d \cdot \delta s$$

$d \cdot r \delta \frac{1}{r}$, $d \cdot \delta \lambda$ and $d \cdot \delta s$ being restrained to mean the differentials of those quantities with regard to $n_i t$ only.

$$\begin{aligned} \delta \cdot r \left(\frac{dR}{dr} \right) &= d \cdot \frac{r \left(\frac{dR}{dr} \right)}{dr} \cdot \delta r + d \cdot \frac{r \left(\frac{dR}{dr} \right)}{d\lambda} \delta \lambda + d \cdot \frac{r \left(\frac{dR}{dr} \right)}{ds} \delta s \\ &= -a d \cdot \frac{r \left(\frac{dR}{dr} \right)}{da} r \delta \cdot \frac{1}{r} + d \cdot \frac{r \left(\frac{dR}{dr} \right)}{dt} \delta \lambda + d \cdot \frac{r \left(\frac{dR}{dr} \right)}{ds} \delta s \end{aligned}$$

$$\delta \cdot \left(\frac{dR}{d\lambda} \right) = -a d \cdot \frac{\left(\frac{dR}{d\lambda} \right)}{da} r \delta \cdot \frac{1}{r} + d \cdot \frac{\left(\frac{dR}{d\lambda} \right)}{dt} \delta \lambda + \frac{\left(\frac{dR}{d\lambda} \right)}{ds} \delta s$$

A similar theorem exists with the quantity $\delta \cdot \frac{dR}{dz}$, and it will readily be seen that all the developments δR , $\delta \cdot r \left(\frac{dR}{dr} \right)$, $\delta \cdot \left(\frac{dR}{d\lambda} \right)$ and $\delta \cdot \left(\frac{dR}{dz} \right)$ may be effected very easily by means of Table II.

Similarly, if δ' denote the variation due to the disturbance of the earth by the moon,

$$\delta' R = -a_i d \cdot \left(\frac{dR}{da_i} \right) r_i \delta \cdot \frac{1}{r_i} - d \cdot \left(\frac{dR}{dt} \right) \delta \lambda_i$$

In dR the terms which arise from

$$-a \left(\frac{dR}{da} \right) d \cdot r' \delta \cdot \frac{1}{r} + \left(\frac{dR}{dt} \right) d \cdot \delta \lambda + \left(\frac{dR}{ds} \right) d \cdot \delta s$$

are multiplied by the small quantity m .

Considering in $r' \left(\frac{dR}{dr} \right)$ and R the terms multiplied by $\frac{a^2}{a_i^3}$,

$$r' \left(\frac{dR}{dr} \right) = 2R, \quad \delta \cdot r' \left(\frac{dR}{dr} \right) = 2\delta R;$$

considering the terms multiplied by $\frac{a^3}{a_i^3}$,

$$r' \left(\frac{dR}{dr} \right) = 3R, \quad \delta \cdot r' \left(\frac{dR}{dr} \right) = 3\delta R$$

Hence the value of $r' \left(\frac{dR}{dr} \right)$ and $\delta \cdot r' \left(\frac{dR}{dr} \right)$ may at once be inferred from R and δR .

I reserve the formation of these developments and of the final equations for determining the coefficients of the different inequalities to another opportunity. These equations are voluminous when all sensible quantities are taken into account; but they are formed with so much facility by means of Table II., that error is not likely to arise in this part of the process. Error is more, I think, to be apprehended in the terms of R multiplied by the cubes and fourth powers of the eccentricities; the rest have been verified by an independent method. See p. 39.

Addition to Table I.

	146	149	150			146	149	150			146	149	150				
1 {	148 147	153 152	154 151	}	1	147 {	1 63	69 3	4 67	}	147	155 {	5 71	83 14	11 90	}	155
2 {	150 149	161 162	162 -146	}	2	148 {	64 1	4 68	70 3	}	148	156 {	72 5	11 89	84 14	}	156
3 {	152 151	147 164	148 163	}	3	149 {	2 65	77 0	8 62	}	149	157 {	6 73	93 12	16 85	}	157
4 {	154 153	165 148	166 147	}	4	150 {	66 2	8 62	78 0	}	150	158 {	74 6	16 86	94 12	}	158
5 {	156 155	167 -173	168 -174	}	5	151 {	3 67	63 9	1 79	}	151	159 {	7 75	87 15	13 91	}	159
6 {	158 157	169 170	178 169	}	6	152 {	68 3	1 80	64 9	}	152	160 {	76 7	13 92	88 15	}	160
7 {	160 159	171 176	172 175	}	7	153 {	4 69	81 1	10 63	}	153						
146 {	62 0	2 -65	66 2	}	146	154 {	70 4	10 64	82 1	}	154						

	161	162			161	162				
{	165 164	{	166 163	}	1	147 {	81 9	10 79	}	147
146 {	8 - 77	-	78 8	}	146	148 {	10 80	82 9	}	148

Addition to Table II.

	146	149	150			146	149	150			146	149	150				
1	{ 147 148	{ 152 153	{ 151 154	}	1	10	{ 165 166	154	153	}	10	64	{ 148	 154	152	}	64
2	{ 149 150	{ 146	{ -146	}	2	11	{ 167 168	156	155	}	11	65	{ 149	 -146		}	65
3	{ 151 152	{ 147	{ 148	}	3	12	{ 169 170	 157	 158	}	12	66	{ 150		146	}	66
4	{ 153 154	{ 148	{ 147	}	4	13	{ 171 172	160	159	}	13	67	{ 151		 147	}	67
5	{ 155 156	{	{	}	5	14	{ 173 174	 -155	 -156	}	14	68	{ 152	 148		}	68
6	{ 157 158	{	{	}	6	15	{ 175 176	 159	 160	}	15	69	{ 153	147		}	69
7	{ 159 160	{	{	}	7	16	{ 177 178	158	157	}	16	70	{ 154		148	}	70
8	{ 161 162	{ 150	{ 149	}	8	62	{ 146	 150	 -149	}	62	71	{ 155			}	71
9	{ 163 164	{ 151	{ 152	}	9	63	{ 147	151	 153	}	63	72	{ 156			}	72

On the Precession of the Equinoxes, supposing the Earth to revolve in a resisting medium.

In my last paper on Physical Astronomy, I gave expressions for the variations of the six constants which enter into the solution of this problem, upon the hypothesis that the body revolves in a medium devoid of resistance.

If we suppose a plane to revolve in a resisting medium, about an axis perpendicular to itself, the resistance of the medium can produce no effect, and the phenomena will only be modified in a slight degree by the friction of the plane surface against the medium. If, however, the inclination of the plane on the axis of rotation differs from 90° , the effect of the resistance of the medium becomes sensible, tending to retard the motion of the plane; the effect being greatest when the axis of rotation is parallel to the plane.

This principle is used in some machines, as in self-playing organs, to regulate the motion by means of a vane, of which the inclination to its axis of rotation can be varied at pleasure.

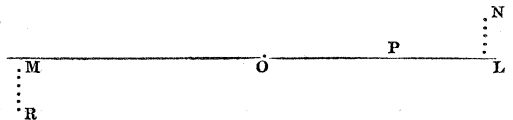
In the case of a sphere, whatever be the direction of the axis of rotation, this effect of the resistance is insensible; and also in the case of a solid of revolution when the axis of rotation coincides with the axis of the figure, but not otherwise. If the difference of the latitude of the axis of rotation from 90° (supposing the equator from which the latitudes are reckoned to coincide with the equator of the figure) be at any time small, the mathematical investigation appears to show, that the effect of the resistance of the medium is to diminish continually this difference. In the case of the earth, this quantity is now insensible; but as the probability is small that this was the case in the first instance, may this circumstance arise from the resistance of a medium of small density acting for a great length of time? and can the change of climate on the surface of the earth, a change of which the probability is indicated by many geological phenomena, be explained in the same manner? It may be remarked, however, that the effect of a resisting medium in diminishing the eccentricities of the orbits of the planets is of the same order, and that these, although for the most part small, are far from having reached zero. The tendency of a resisting medium is also to diminish the major axes of the orbits of the planets; these effects, if they exist, will probably be most sensible

in the case of comets, not only on account of their great eccentricity, but also on account of their small density, in the same manner as a flock of any light substance is wafted by the gentlest wind and prevented from reaching the ground. The eccentricity of the orbit of the comet of HALLEY in 1759 is known with great accuracy, and as its perturbations have been calculated with great care by MM. DAMOISEAU and DE PONTECOULANT, the eccentricity which it should have in 1835, when it will again visit this part of space, unless it be affected by a resisting medium, is also known with great precision. It is scarcely probable, however, that any change will be perceptible in one revolution, even if the cause exists; but the succeeding revolutions of this body will no doubt throw light upon this question. The ratio of the change of the semi-major axis to the change of the eccentricity, due to the action of the resisting medium, is known, being a function of the eccentricity, and independent of the constant, which depends upon the density of the medium; this ratio therefore may also tend to elucidate the question, if it can be determined by observation with sufficient accuracy.

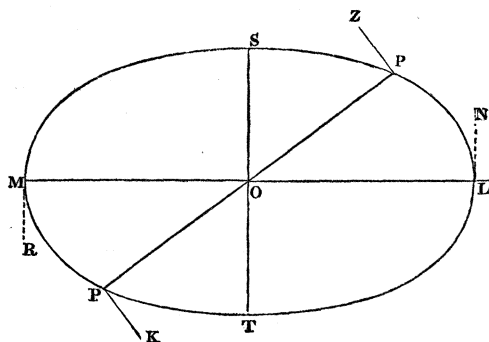
Let x', y', z' be the co-ordinates of any point P corresponding to the elementary portion of the surface ds , and referred to axes passing through the centre of gravity and revolving with the body in motion.

Let P be the point of which the co-ordinates are x', y', z' , AP the direction of the normal at the point P, BP perpendicular to the axis of instantaneous rotation, and cutting it in B, and CP the direction of motion of the point P. I suppose the resistance of the medium to create a force proportional to $v^2 \cos APC ds$, acting in the direction of the normal AP upon the point P, v being the velocity of the point P.

Suppose the straight line M O P L to revolve about an axis passing through O, and perpendicular to it, and in the direction LN, the action of the resisting medium will be in the direction NL, on one side only of the line OL, upon all the points P between O and L, and upon all the points between MP it will be in the contrary direction RM, and on the other side of the line.



Now, let $LSMT$ be the section of a cylinder revolving about an axis, passing through O perpendicular to the plane $LSMT$, and let the cylinder revolve in the direction LN . The action of the resisting medium will be in the direction ZP , perpendicular to OP upon all the points P between LS ; and in the contrary direction KP upon all the points,



P between TM . These remarks show that in what follows, the integrations must not be made throughout the whole surface of the body revolving: this consideration however does not affect the nature of the results.

The equation to a plane perpendicular to the axis of rotation, and passing through the centre of gravity of the body, is $px + qy + rz = 0$.

Let the body revolving be a spheroid of which the equation is

$$x^2 + y^2 + z^2 (1 + e^2) = a^2 (1 + e^2)$$

The equation to the tangent plane to the spheroid at the point x, y, z is

$$xx' + yy' + zz' (1 + e^2) = a^2 (1 + e^2)$$

The equations to the planes from whose intersection the line PC results, are

$$\begin{aligned} * z(qz' - ry') + y(rx' - pz') + z(py' - qx') &= 0 \\ px + qy + rz &= D \end{aligned}$$

D being a constant. The equations to the line PC are

$$\begin{aligned} x\{r(qz' - ry') - p(py' - qx')\} + y\{r(rx' - pz') - q(py' - qx')\} &= 0 \\ x\{q(qz' - ry') - p(rx' - pz')\} + z\{q(py' - qx') - r(rx' - pz')\} &= 0 \end{aligned}$$

and neglecting p^2, q^2, pq ,

$$\begin{aligned} x(qz' - ry') &= y(pz' - rx') \\ x(qy' + px') &= z(pz' - rx') \end{aligned}$$

The equations to the direction of motion of the point P are

$$\begin{aligned} x(pz' - rx') &= y(ry' - qz') \\ x(qx' - py') &= z(ry' - qz') \end{aligned}$$

Cos. angle, which the direction of motion of P makes with the normal to the surface or $\cos APC$

$$= \frac{x'(ry' - qz') + y'(pz' - rx') + z'(1 + e^2)(qx' - py')}{\sqrt{\{(ry' - qz')^2 + (pz' - rx')^2 + (qy - px')^2\}} \sqrt{\{x'^2 + y'^2 + z'^2(1 + 2e^2)\}}}$$

* The notation is the same as p. 20, except that the accents at foot of x, y, z , are omitted.

$$= \frac{e^2 z' (q x' - p y')}{r \sqrt{x'^2 + y'^2} \sqrt{x'^2 + y'^2 + z'^2}} \text{ nearly.}$$

The resistance acting in the direction of the normal, and since the velocity
 $= \sqrt{x'^2 + y'^2} \sqrt{p^2 + q^2 + r^2}$ nearly;

$$C dr = 0$$

$$B dq + (A - C) r p dt = dt \int \frac{\{z' x' - x' z' (1 + e^2)\} e^2 z' (q x' - p y') \sqrt{x'^2 + y'^2} ds (p^2 + q^2 + r^2)}{r \{x'^2 + y'^2 + z'^2\}}$$

$$A dp + (C - B) q r dt = dt \int \frac{\{y' z' (1 + e^2) - z' y'\} e^2 z' (q x' - p y') \sqrt{x'^2 + y'^2} ds (p^2 + q^2 + r^2)}{r \{x'^2 + y'^2 + z'^2\}}$$

$$\sin \frac{C - A}{A} (nt + \gamma) dc + c \frac{(C - A)}{A} \cos \frac{C - A}{A} (nt + \gamma) d\gamma$$

$$= -\frac{n dt e^4}{A} \int \frac{x' z'^2 (q x' - p y') \sqrt{x'^2 + y'^2} ds}{\{x'^2 + y'^2 + z'^2\}}$$

$$\cos \frac{C - A}{A} (nt + \gamma) dc - c \frac{(C - A)}{A} \sin \frac{C - A}{A} (nt + \gamma) d\gamma$$

$$= \frac{n dt e^4}{A} \int \frac{y' z'^2 (q x' - p y') \sqrt{x'^2 + y'^2} ds}{\{x'^2 + y'^2 + z'^2\}}$$

$$\text{since } \int x'^2 z'^2 ds = \int y'^2 z'^2 ds$$

$$dc = -\frac{n dt e^4 c}{A} \int \frac{x'^2 z'^2 \sqrt{x'^2 + y'^2} ds}{\{x'^2 + y'^2 + z'^2\}} + \frac{n dt e^4}{2A} \sin 2 \frac{(C - A)}{A} (nt + \gamma) \int \frac{x' y' z'^2 \sqrt{x'^2 + y'^2} ds}{\{x'^2 + y'^2 + z'^2\}}$$

neglecting the term which is periodic,

$$dc = -nc \frac{e^4 dt}{A} \int \frac{x'^2 z'^2 \sqrt{x'^2 + y'^2} ds}{\{x'^2 + y'^2 + z'^2\}}$$

$$\text{Let } \int \frac{x'^2 z'^2 \sqrt{x'^2 + y'^2} ds}{\{x'^2 + y'^2 + z'^2\}} = D$$

D being a positive quantity.

$$dc = -\frac{n D c e^4 dt}{A} \quad e^{\frac{1}{c}} = \frac{n D e^4 t}{A}, \quad e \text{ being the base of Naperian logarithms.}$$

When t is infinite $c = 0$; hence the latitude of the axis of instantaneous rotation increases until it reaches 90° , which is its limit.

Having determined the variations of c , γ and n by means of the above equations, the variations of the other constants ω , ψ_0 and ϕ_0 may be determined from the equations

$$p dt = \sin \phi \sin \theta d\psi - \cos \phi d\theta$$

$$q dt = \cos \phi \sin \theta d\psi + \sin \phi d\theta$$

$$r dt = d\phi - \cos \theta d\psi$$